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Errores de estudiantes de primaria en la resolución de problemas con fracciones.

Primary student mistakes in solving problems with fractions.

Iván Andrés Padilla-Escorcia[a], Yesika Paola Rojas-Sandoval[b], Silvia Palencia-Quiroz[c]

[a]Master's Degree in Education, iapadilla@mail.uniatlantico.edu.co, https://orcid.org/0000-0003-1210-3712, Universidad del Atlántico, Usiacuri, Colombia

[b]Master's Degree in Educational Informatics, yesikarojas@mail.uniatlantico.edu.co, https://orcid.org/0000-0003-0460-2335, Universidad del Atlántico, Barranquilla, Colombia

[c]Bachelor in Mathematics, srpalencia@mail.uniatlantico.edu.co, https://orcid.org/0000-0002-6101-7848, Universidad del Atlántico, Barranquilla, Colombia.

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Palabras claves

Errores, Escuela, Estudiantes De Primaria, Fracciones, Solución de Problemas.

Resumen: El objetivo de la investigación consistió en caracterizar los principales errores que presentan los estudiantes de tercer grado de primaria en la resolución de problemas con fracciones Método: se empleó un enfoque cualitativo con diseño de estudio de caso, determinado por tres fases (preactiva, interactiva y postactiva) en los cuales se recolectó la información a través de observaciones no participantes, trabajo de campo, prueba diagnóstica y entrevistas semiestructurada a los estudiantes. Los principales resultados evidencian que los estudiantes no cuentan con nociones definidas ni procesos metacognitivos de autorregulación en la resolución de problemas con fracciones, puesto que presentan dificultades para relacionar el lenguaje cotidiano con lenguaje matemático. Se concluyó que los errores presentados por los estudiantes de tercer grado se dan debido a debilidades en las habilidades de análisis, razonamiento e interpretación en la lectura de un problema de contexto.

*Autor para correspondencia:iapadilla@mail.uniatlantico.edu.co

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Keywords

Mistakes, School, Elementary Students, Fractions, Problem Solving.

Abstract: The objective of the research was to characterize the main errors presented by third grade elementary school students in solving problems with fractions Method: a qualitative approach with a case study design was used, determined by three phases (preactive, interactive and postactive) in which information was collected through non-participant observations, field work, diagnostic test and semi-structured interviews with students. The main results show that students do not have defined notions or metacognitive processes of self-regulation in solving problems with fractions, since they have difficulties in relating everyday language with mathematical language. It was concluded that the errors presented by third grade students are due to weaknesses in the skills of analysis, reasoning and interpretation in the reading of a problem in context.

Introduction

In the educational context, it is common to hear comments about the difficulty of learning mathematics. Components linked to affection, emotions, attitudes or beliefs have a great influence on the learning of this area of knowledge. Similarly, health, the family context and the socioeconomic situation associated with the educational system itself jointly or particularly, implies that students do not feel motivation or attraction to learn mathematics (Ricoy and Couto, 2018; Padilla et al; 2018).

Now, within the purely mathematical field, children and young people in school education present difficulties in solving problems and applying the contents of mathematics in everyday situations (Arias and Oviedo, 2020). Undoubtedly, it permeates up to tertiary education when they start their higher education careers (Murcia & Henao, 2015). In this sense, capturing the attention of students often depends on the different stimuli that are developed in the classroom. In turn, it generates challenges that teachers must face in their pedagogical practice as an object of teaching-learning of mathematics. And what happens when both teachers and students interact with each other. Therefore, enhancing the teaching of mathematics implies strengthening this interaction (Montes, Gamboa & Fernández, 2013).

Similarly, the low performance of students in the area of mathematics occurs frequently, causing negative perceptions in the community. This being evident in the analysis made by Ayala (2015), who sustains that in Colombia the different National and International evaluations carried out in different years, low results have been obtained that denote a minimum effort of the Country to comply with the necessary standards of international mathematical competences. Likewise, the Colombian Institute for the Evaluation of the Quality of Education (ICFES) reported in the 2018 saber 11° tests an average score of 52 in mathematics, which groups 1% of the students who solved the test in level 1 and 10% in level 2% (ICFES, 2019).

Thus, in the teaching of mathematics, it is common to find situations in which students, when developing an activity of a certain topic, do not have sufficient mastery of it (inadequate cognitive scheme), which prevents them from developing mathematical procedures correctly. Since these errors intervene in the reinforcement of the different contents, it is essential that both teachers and students recognize them and assume the precise role to overcome them, in order to obtain learning achievements that contribute to the provision of strategies that strengthen learning, specifically in those aspects that generate more difficulties (Engler et al., 2004).

In this way, Rico (1995) affirms that errors in mathematics are immersed in the learning process of students and are reflected in the different activities

they perform, providing the results obtained for an improvement in the teaching processes. Thus, he classifies the most common errors in mathematics in five categories:

1. Errors due to language difficulties: this category emphasizes everything related to symbols, concepts and vocabulary of the different mathematical topics that may represent for students' confusions caused by an inadequate assimilation in their learning process (Rico, 1995; Engler et al 2004). A very particular case with this type of error is when the student is presented with a problem and does not understand exactly the terms used in the statement, since he does not understand what it says and therefore does not know what to do, a situation that makes teachers think that the student fails to solve problems, ignoring that the problem lies in the understanding of concepts and vocabulary.

2. Errors due to difficulties in obtaining spatial information: it is complex for students to think through spatial or visual images (Rico, 1995). This means that, at the moment of observing a graph, scheme or figure from different perspectives, there are inconveniences to give an interpretation of it in any type of problem situation.

3. Errors due to a deficient learning of facts, skills and previous concepts: implies the lack of necessary knowledge that allows approaching a mathematical activity appropriately, being fundamental the clarity in the different logical procedures that can be performed and about the contents to be used (Rico, 1995; Martínez, 2015). Given the above, it can be presented that in order to solve a problem, a third-grade student performs a division, but it turns out that in the place of the dividend the students places a number smaller than the divisor. What leaves in evidence that the student has a knowledge that is not proper of division, performing an inadequate procedure and also shows shortcomings in the concept of division that he is supposed to have clear.

4. Errors due to incorrect associations or rigidity of thought: they are produced by not appropriating the topics with different situations, either new or similar, so that the student makes an error when modifying the way of performing some activities, i.e., that a problem situation that has been worked previously in addition, in a new occasion is used but with multiplication and the student has not yet understood that the operation has already been changed due to the lack of flexibility in his thinking (Rico, 1995; Del puerto, 2004; Minnaard, 2004).

This type of rigidity includes perseveration errors in which single elements prevail; association errors that are evidenced when students have some type of incorrect reasoning or make associations that are not appropriate in any task; interference errors between concepts or operations; assimilation errors, where a type of fault or distraction interferes with the way in which the student manages to perceive the information and negative transfer errors from previous tasks, in which the analysis of errors in a set of exercises is acquired (Rico, 1995; Engler et al 2004).

5. Errors due to the application of irrelevant rules or strategies: this can occur in relation to the teachers' lesson planning, where they use new strategies or strategies similar to those they normally use and that at some point were successful in other content; however, this situation tends to generate confusion that leads them to commit this type of errors and in turn interfere in their learning process (Rico, 1995).

Now, taking into account that students can make mistakes when confusing various rules of mathematics, the rules of arithmetic operations and fractions are the most difficult content for students to understand (Fazio and Siegler, 2011). This is mostly due to the fact that students look at fractions as a

whole or as any other natural number, forgetting that they are made up of a numerator and a denominator that cannot be treated as separate entities (Butto, 2013).

In addition to this, research by Sartono and Karso (2020), Bottge et al. (2020), show that the main difficulties of students in learning fractions lie in not knowing how to convert fractions to mixed fractions, multiplication of fractions and operations that combine fractions with whole numbers, which in turn leads to not knowing how to solve a problem that brings together the aforementioned elements. In this sense, Martínez and Lazcano (2001) describe fractions as a part-whole relationship, therefore, in the teaching of fractions, it is necessary to perform actions on a whole (unit); once it has been torn, cut, sliced, divided and colored or marked in equal parts, or otherwise it is assumed as if it were, through oral, written, graphic and/or spoken languages. The need arises for the situations in which the utility of fractions in the real context is framed to be of this type. Thus, the understanding of fraction depends on the understanding of each of its meanings, being important to have clarity in each of them, since fractions have multiple interpretations: as ratio, quotient or division (González, 2015).

Thus, according to González (2015), the main errors in learning and solving problems with fractions are determined by the number of meanings that these have, as well as the language or diagnosis that students have about this field and the complexity of mathematical concepts, in addition to the fact that learning fractions assumes a broad and flexible course in which students have been able to understand the subject. This is confirmed by the depth of these contents in the curriculum.

In view of this, there is a need for teachers to take dynamic positions regarding the teaching they offer to their students, since traditional teaching, which is directed using only the board and markers, does not allow them to have a different vision of the topics, without understanding the concept through daily life situations in which they can be evidenced and without taking into account practical knowledge.

On the other hand, research conducted by Pochulu and Rodriguez (2012) describe actions that a mathematics teacher should take into account in the elaboration of strategies in the teaching-learning processes of mathematics contents especially in the topic of fractions in a class. These can be edited based on novelties and experiences that are evidenced in practice, so that they can be adapted according to the needs of the students. Likewise, the way in which teachers teach the different mathematics subjects is influential in the assimilation and comprehension of new knowledge by students. Since competencies are evaluated and developed from the resolution of tasks that students perform, thus, the design of good tasks allows the development and evaluation of competencies and skills of learners (Pochulu, Font, & Rodriguez, 2016).

In this sense, activities that cause students to reflect on their results are of great importance, since the recognition of their faults could contribute to their achievement in school. Mistakes should be means by which students come to understand them as one more challenge that they must overcome in the course of their learning process if they wish to succeed in life (MEN, 1998). This shows the need to recognize errors in order to think of the most appropriate strategies to overcome them, always remembering that it is something natural at any stage of learning that should not be feared or hidden from teachers, since otherwise they would not contribute to the process of knowledge construction.

Thus, the objective of this article covers all the errors described above and specifically to characterize the main errors of elementary school students when solving problems with fractions in educational institutions in Barranquilla (statement of

the problem, development of background, purposes and rationale of the research).

Method

This research is developed with a case study design, based on a qualitative approach, from the statement of an observed problem it is possible to develop descriptions, studies and interpretations that allow generating new theories, making use of techniques for data collection that take into account the perspectives and context of the participants (Sampieri, 2014).

The methodology used consists of three phases taken from Pérez and Martínez (cited by Álvarez 2012) that show the content of the techniques and instruments to be used. The first phase is called pre-active, which seeks to obtain information on the problems presented by the students in the fractions and to analyze the way in which they are facing the learning of fractions, based on the above, the non-participatory observation and diagnostic test applied to the students are taken into account; The second is the interactive phase, which allows detailing the results obtained from the previous phase on the problem studied, allowing to establish the relationship between the difficulties that affect the development of activities with fractions that lead them to make mistakes, in this phase a structured interview is applied to students to collect the required information. Finally, there is the post active phase where a text analysis matrix is made to organize the characteristics of the errors presented most frequently by the students, based on the details that can be found in the texts with which they work.

In this order of ideas, the selected population were third grade students belonging to the Institución Educativa Fundación Pies Descalzos Centro Comunitario Barranquilla, which have three teachers assigned for each of the grades A, B and C. On the other hand, for the selection of the sample, the case study design was taken into account considering Padua (1979) who refers to the fact that the intentional samples are the product of the choice according to the criteria of the expert, who selects some cases that turn out to be characteristic; so that 2 students are chosen, one with good academic performance and the second with a regular academic performance in comparison with the other, taking into account that they are students who stand out in the area and that in this way the behavior of the rest of the population can be evidenced from the possible weaknesses found in them.

Subsequently, the techniques and instruments that are useful for the fulfillment of the objective of this research and its purpose were the non-participatory observation, the diagnostic test composed of 10 questions focused on the subject of fractions and carried out individually during a time limit of 60 minutes; it is important to mention that the test was validated by means of the judgment of experts and was only taken as an object of analysis. Secondly, the structured interview directed to the students was conducted in a time limit of 30 minutes with 8 questions related to the inconveniences they had in the test and the way in which they conceive the subject in general.

Finally, a text analysis matrix is used from the books used by the institution (Matemáticas al máximo, first edition, 2015), one with definitions - examples and the second, practice exercises for students, this in order to analyze their content and observe the influence that the approach to concepts, properties, examples or sequence has on the presence of errors in students with respect to notation and writing that prevents them from fully understanding the topic of fractions.

Additionally, a triangulation is performed that will allow enriching the results of the research, giving greater reliability, accuracy and consistency to the same taking into account the observation, diagnostic test and interview directed to students,

based on Hernandez S. (2014) who says that data triangulation is that which allows the use of different processes to interpret the information.

Results

Diagnostic test

Table I.: Answers To Item 1.

Student 1

1. **Define con tus palabras lo siguiente.**

¿Qué es una fracción?

una fracción es que tiene denominador y numerador

¿Qué indica el numerador de una fracción?

una fracción heterogénea

¿Que indica el denominador de una fracción?

una fracción homogenica

Student 2

1. **Define con tus palabras lo siguiente.**

¿Qué es una fracción?

una fracción es lo que indica un numerador y un denominador

¿Qué indica el numerador de una fracción?

El numerador es el que indica los partes que tomamos.

¿Que indica el denominador de una fracción?

El denominador indica las partes que nos quedaron.

In the first question of the test, it was identified that students arrive at a similar answer, however, they do not express a grounded concept of what a fraction is (Table I). Both students show clarity regarding what the elements in a fraction indicate, so it is expected that when they recognize them, they know what each one indicates, however, in one of the cases there is confusion with other definitions in such a way that their arguments, as Godino (2004) states, are not valid and thus they make mistakes associated to language difficulties, since they present problems in understanding mathematical concepts and vocabulary.

Table II. Answers To Item 2.

2. **Observa las siguientes imágenes y completa.**

El círculo está dividido en 3 partes iguales. Cada parte representa 1/ del círculo.

El rectángulo está dividido en 6 partes iguales. Cada parte representa 5/ del rectángulo.

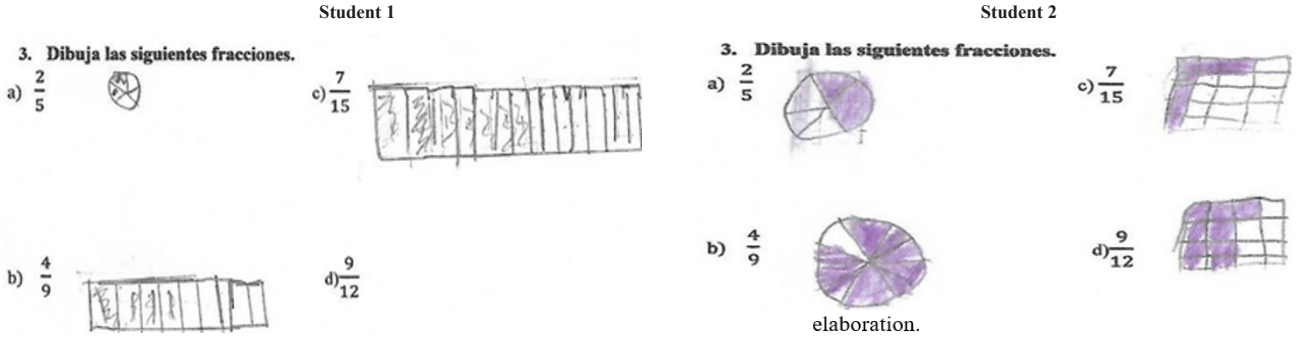
2. **Observa las siguientes imágenes y completa.**

El círculo está dividido en 3 partes iguales. Cada parte representa 2/ del círculo.

El rectángulo está dividido en 6 partes iguales. Cada parte representa 5/ del rectángulo.

In this case, both students 1 and 2 present difficulties in interpreting and analyzing graphic representations that show the concept of fraction in a geometric figure such as a circle or rectangle, as shown in Table II. This shows difficulties in the development of spatial thinking skills, as they do not process the information generated by the drawings, something that in mathematical tasks is very common to find from spatial or visual images (Rico, 1995). Thus, the error is associated with language difficulties due to the lack of understanding of the subject in relation to Table I previously explained in relation to the error due to difficulties in obtaining spatial information.

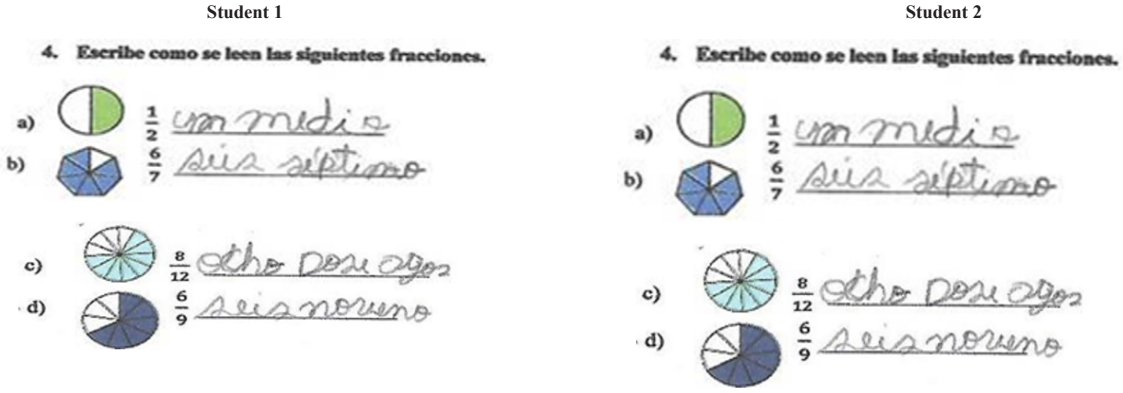
Table III. Answers To Ítem 3.



elaboration.

In the previous figure, Table III, we can see errors associated with difficulties in language, obtaining spatial information that have been previously evidenced (Table 1 and 2) and assimilation errors, since the representations that are made do not correspond to the indicated fraction (student 2), this is due to not having clarity about the elements of a fraction related to mathematical language, as well as weaknesses in reading, writing and problems in deciphering, demonstrating or explaining through images due to failures in information processing. On the other hand, student 1 performs 3 of the 4 representations, however, he is affected by assimilation and language errors due to the lack of understanding of the content by not dividing the unit into equal or disproportionate parts.

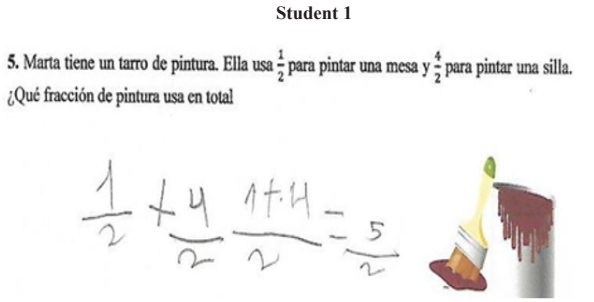
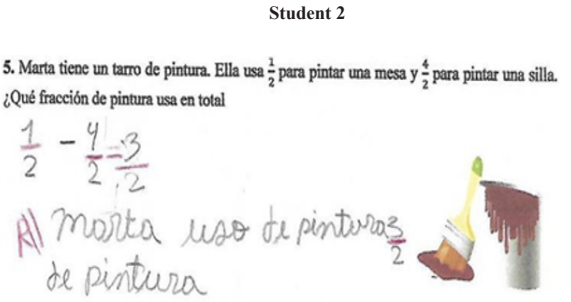
Table IV. Answers To Item 4.



In item four, Table IV, it can be seen that students present difficulties in terms of mathematical language and assimilation errors that occur due to incorrect associations regarding the writing of fractions, since these

change when they are numbers greater than ten and cause rigidity in the model of thought with which the student is initially rooted, which causes the student to have certain difficulties in adapting to new situations (Rico, 1995). However, it should be noted that although they present difficulties in the way of writing fractions (spelling errors), they have knowledge of how they should be named, which is mainly related to errors in mathematical vocabulary and those mentioned initially.

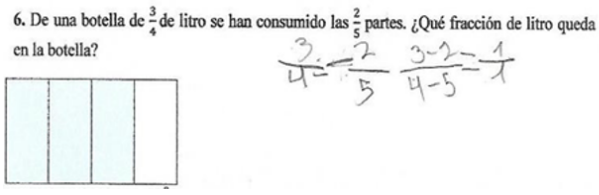
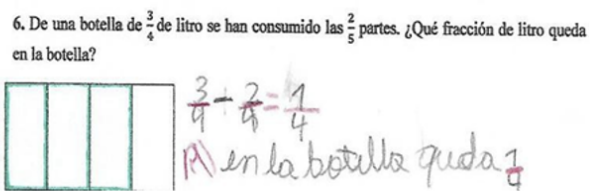
Table V. Answers To Item 5.

Student 1	Student 2
	

In this question, it is possible to evidence (table 5) that student 1 has a mathematical process that takes into account the guidelines required in the situation, in spite of not giving a conclusive answer, he manages to give a solution to the problem, showing one of the purposes of mathematical practices (Godino, 2004) such as reasoning and problem solving.

However, based on the previous results, there are deficiencies in the mathematical language that imply an absence in the vocabulary, causing this type of cases in which the student is not able to establish a final analysis. From student 2, it is observed in table V. that there is an inadequate process in the operation to be performed, so it can be inferred that he presents errors caused due to deficient learning in previous concepts and notions that prevent him from an appropriate interpretation in the situation, this is due to the lack of understanding about knowledge regarding an addition or subtraction that he has had to handle previously. Likewise, confusions when executing rules, properties or symbols that he/she does not master and that lead to this type of errors (Rico, 1995).

Table VI. Answers To Item 6.

Student 1	Student 2
	

For this case, in Table VI, the two students make errors regarding operations between heterogeneous fractions, which from school mathematics is not an adequate process (Godino, 2004). It can be observed that, in spite of having determined the operation to be used, they did not take into account the type of fraction presented by the problem, thus presenting errors due to difficulties in language caused by the scarcity of previous notions in the students and also errors due to rigidity in thinking, because when changing the way of operating, confusions arise that influence the processes to be carried out.

Table VII. Answers To Item 7.

Student 1	Student 2
7. María se ha gastado $\frac{2}{7}$ del dinero que le dieron de paga sus abuelos en comprar un libro de aventuras. También gastó $\frac{1}{7}$ en una bolsa de dulces. La cantidad que le dieron sus abuelos fue $\frac{5}{7}$ de la paga. ¿Qué fracción le ha quedado a María de la paga?	7. María se ha gastado $\frac{2}{7}$ del dinero que le dieron de paga sus abuelos en comprar un libro de aventuras. También gastó $\frac{1}{7}$ en una bolsa de dulces. La cantidad que le dieron sus abuelos fue $\frac{5}{7}$ de la paga. ¿Qué fracción le ha quedado a María de la paga?
	

In Table VIII, students performed the situation presented to them using different strategies for its solution; however, there are errors due to the execution of arithmetic rules and fractions, which are the most complex for them (Fazio and Siegler, 2011). They also presented errors due to language difficulties, since in the execution there are processes that are not used adequately due to lack of mastery of symbols and concepts.

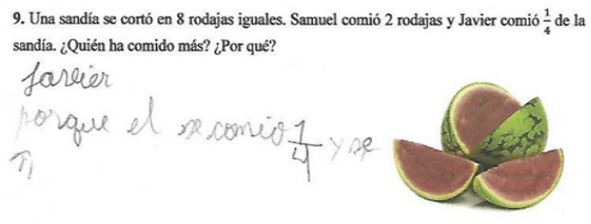
Table VIII. Answers To Item 8.

Student 1	Student 2
8. Observa la siguiente imagen:	8. Observa la siguiente imagen:
	
Cuenta la cantidad de mariquitas que hay y repártelas en ocho grupos iguales, de tal manera que cada grupo represente $\frac{1}{8}$ del total. ¿Cuántas mariquitas son $\frac{3}{8}$ de 16?	Cuenta la cantidad de mariquitas que hay y repártelas en ocho grupos iguales, de tal manera que cada grupo represente $\frac{1}{8}$ del total. ¿Cuántas mariquitas son $\frac{3}{8}$ de 16?
	

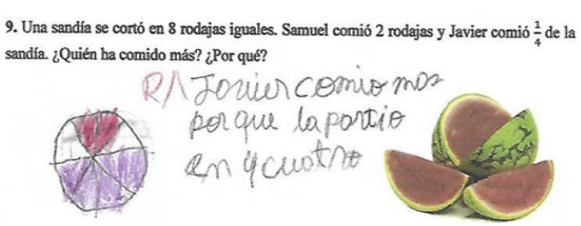
In this item, the students did not manage to solve the problem of table VIII. In the case of student 1, he tries to distribute what the situation asks for but does not keep in mind that he must assemble eight equal groups, showing errors due to incorrect associations or rigidity in the way of thinking, since he has been able to infer his solution to any other situation that he has done previously due to the lack of flexibility in his thinking (Rico, 1995; Del puerto, 2004; Minnaard, 2004). Likewise, they present reading, interpretation and content comprehension problems, which together give way to errors due to the lack of understanding of mathematical language; it should be noted that for many students the analysis of certain types of situations or perceptual syntheses imply more work, which leads them to present difficulties and produce errors (Rico, 1995).

Table IX. Answers To Item 9.

Student 1



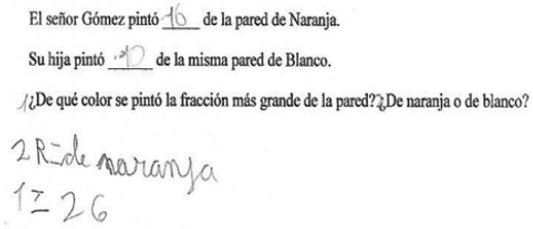
Student 2



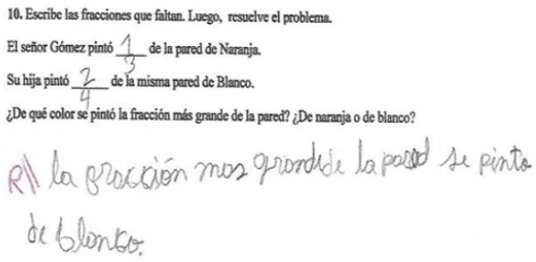
The students showed an answer without arguments and justification, Table IX. Once again, they show errors due to language difficulties in relation to the comprehension of the contents with little mastery of the rules, symbols, properties and mathematical operations, which are the cause of the inadequate interpretation of the statements. However, sometimes many of the errors presented by students are not only due to a lack of knowledge of the subject, but also to the misuse of these errors in identifying the mathematical operation that should be used to solve an application problem with fractions (Godino, 2004).

Table X. Answers To Item 10.

Student 1



Student 2



In the last item of the test, table X., student 1 places natural numbers instead of fractions as required in the statement, where operations and concepts of fractions are interfering with others, because there are mistakes in reading, writing and a wrong impression that perhaps the student has had when finding similarities in other exercises and therefore anticipates, The student solves quickly without taking into account what has been worked on throughout the test, thus presenting errors of assimilation, interference, association and negative transfers where the student at the time is not being aware that is making a mistake for not understanding the meaning of the concepts used. This may be due to "the experience of previous similar problems can produce a rigidity in the habitual way of thinking and a lack of flexibility to encode and decode new information" (Rico, 1995, p.89).

Semi-structured interview with students

In the interview directed to the students, questions related to the notions of fractions are used to see how the students' knowledge of fractions is in order to compare them with what is presented in the test, as well as questions about the way in which they approach the different problem situations, the way in which they develop and the type of difficulty they encounter when solving them; On the other hand, questions were asked about motivational aspects regarding the activities established in the institution and thus, from their answers, determine

those difficulties that may be of influence or that are the cause of the errors evidenced in the test. In this space, it was possible to observe the students' reactions to each of the questions posed, noting those particular data that allow a broader analysis in relation to the written answers.

Within the results obtained from the interview, it can be found that students come to present certain notions about fractions, since they manage to identify them, but there is no understanding regarding their concepts, elements and representation. Being more specific, students state that they recognize the elements of a fraction, but when they are asked to mention them they do not know what they are, so that there is no association between what the fraction represents and its terms, but there is a separation that shows that because they do not understand the concept properly they do not make good use of them, so that sometimes they implement it correctly and sometimes they do not even identify whether its application is valid (Godino, 2004).

Thus, in the first questions, difficulties related to mathematical contents were identified, which correspond to the lack of mastery of these contents and cause students to make mistakes with arguments, actions or procedures that are not valid; in addition, in relation to the results of the diagnostic test, this difficulty would be generally causing one of the most frequent errors in students, which is associated with difficulties in language and mathematical vocabulary.

Likewise, in the questions that allow to deduce the way in which they approach problems, it was found that it is difficult for them to understand what the statement poses them without mentioning the confusion that the operation that allows them to perform the process to reach the solution of the same, also confusing the procedure in each of the basic operations; which means that what the students know is not the necessary knowledge to be able to learn a new content (Godino, 2004). Thus, it is in this sense that difficulties related to the lack of mastery of previous content are externalized and that, despite having clarity in some aspects of the new, there is little understanding of the processes that are performed based on justifications that are not valid from the point of view of school mathematics and that are associated with difficulties related to mathematical content.

In this order of ideas, the students are asked about how they feel in the classes to see if there is some type of factor that may be influencing the learning of the subject matter and consequently, They like the strategies that the teacher uses because they are cheerful. However, according to what was evidenced in the test, these strategies are not being significant for the students and it may be due to a bad structuring that causes confusion in them, which makes them unable to assimilate the concepts in an adequate manner and the mistakes made are due to an inadequate understanding of the subject matter as could be evidenced in the diagnostic test, so that they cannot understand the purpose of mathematics (Godino, 2004).

Additionally, in the non-participatory observation, students were easily distracted, due to different factors that can be generated by schedules, the number of classmates, the availability of resources, or the objects found inside the classroom, which is sometimes overloaded, so that these situations caused difficulties caused by the sequentially of activities that originate in the organization of the educational center and that affect the difficulties related to mathematical content.

Text analysis matrix

Table XI. Text Matrix.

Author(s)	Year	Name of textbook	Edition	Editorial
Adapted from the Primary Mathematics project Ministry of Education, Singapore.	2015	Matemáticas al máximo Cuaderno de práctica	1°	Scholastic Education International (Singapore)
Adapted from the Primary Mathematics project Ministry of Education, Singapore.	2015	Matemáticas al máximo Texto del Estudiante	1°	Scholastic Education International (Singapore)

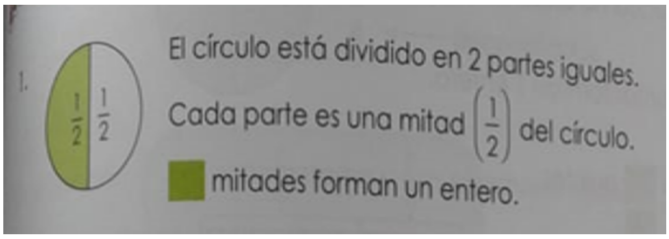


Figure 1. Mathematics book section

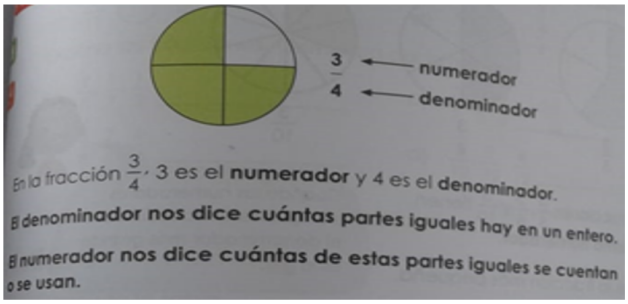


Figure 2. Mathematics book section

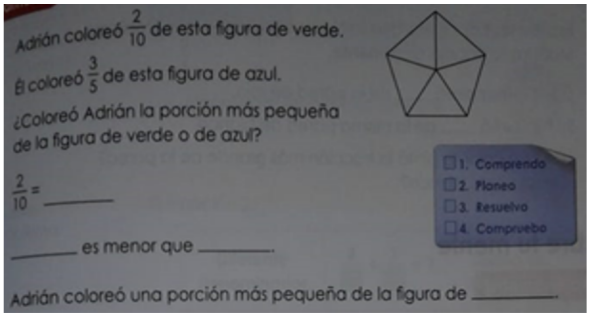


Figure 3. Mathematics book section
Source: ¡Matemáticas al máximo! book

Some findings:

Based on the previous images, several aspects were found that may cause incident difficulties in the students' errors, since they are the texts used by the teachers in the subject of Mathematics and are taken as support guides. In Figure 1 and 2 it can be observed that there are examples related to the topics worked with the students, however, different terms are used than those exposed in class, which causes confusion at the moment of an autonomous work and assimilation errors due to reading problems, when the teacher explains with different techniques from those found in the book and when they have to do exercises or want to study from the book, they find that the explanations are approached in a different way and cause incorrect associations reflected in the diagnostic test, such as considering fractions as a whole or as any other natural number because they do not understand the term used (table II). Likewise, they do not provide a broad concept of what a fraction is, as a whole part, as a quotient, operator or ratio; without addressing the meanings that are so important for the learning process of the students because by not understanding their meaning and the relationships between them, significant progress is not achieved, since there must be clarity to be able to make the relevant interpretations in problem situations related to their meanings (Hincapié, 2012).

Based on the above, the problem situations posed in the practice text do not favor to a great extent the understanding of mathematical content, since they specify the step by step to reach a solution, generating limits for student learning, in turn preventing the development of skills in communication, reasoning and problem solving to be used adequately.

Based on the analysis of the previous texts:

- By not clearly providing concepts and examples: mathematical content difficulties are generated in students that lead them to make mistakes due to difficulties in mathematical language, where they do not understand symbols, properties, terms, vocabulary, among others; and that are used incorrectly, without any kind of validity. The above causes errors with respect to the semantic part of mathematical language that influence when students make mistakes related to the management of concepts, contents and procedures.

- By specifying the steps to solve a problem: it prevents students from finding the way to solve them, so they make incorrect procedures by applying techniques of which they do not have adequate knowledge, ignoring some algorithms and necessary concepts that cause errors due to deficient prior learning.

Triangulation of information

Table XII. Triangulation Of Information: Observation, Diagnostic Test And Interview.

Categories	Observation	Diagnostic Test	Interview	Triangulation
Errors in solving fraction problems	Students in problem-solving activities always took a long time to respond, were confused and did not express themselves adequately. At the beginning of the diagnostic test, they were enthusiastic and intrigued, and throughout the test they were distracted by items they were working with and by striking objects in the classroom. The teachers in the explanations do not have complete attention of the students and do not take the errors as something important in the different activities that they assign, in addition they do not know the types of errors that exist.	The students' answers in items related to concepts, representation and elements of fractions are erroneous, they have faults with respect to reading and writing. In problem solving they fail to establish the appropriate operations and, in some cases, they accommodate the operation as they see fit, having procedures that are initially good and then erroneous. Consequently, students use vocabulary inadequately, concepts interfere with each other and they do not master the previous concepts that are the basic operations.	The students express difficulties in the problem situations when they were unable to interpret what the exercise proposed, in addition to the operation and procedure that they had to take into account. They consider that the teachers use strategies that allow them to see the fractions in an easy way, however, they reflect weaknesses in the concepts, which means that the strategies are not being significant to have the student's full attention. Both students recognize what a fraction is but find it difficult to give explanations regarding what it means.	The data collection shows similarity in the analysis and it is remarkable the relation of the difficulties in the errors presented by the students in the resolution of problems with fractions. There are no contradictions and they tend to the same information, so that not only the contents influence, but also the importance that teachers give to the errors in the activities performed by the students and their possible causes as part of recognizing their existence and classification.

Discussion

According to the errors evidenced by the students regarding the solution of problems with fractions, it is important to highlight that they do not understand the meaning represented by each of the parts of a fraction, which prevents them from obtaining spatial information, in turn, they make mistakes related to the reading and pictorial representation of fractions. Presenting difficulties in proceeding to add or subtract heterogeneous fractions, additionally they have inconveniences in establishing equivalences between fractions, which is worrisome if one takes into account that, it is expected that students at this level of schooling consolidate the concept of fraction, so that in basic secondary school the extension of numerical sets is adequately deepened, with which they will access the learning of rational numbers (Castro, 2017).

Likewise, these results show that in schools, the concept of fraction is not approached in a correct way since, in general, it is only seen as the distribution in equal parts of an object, which in many occasions limits that beyond this, students apply these in contextual situations, a big problem in this research where there was little analysis, interpretation and understanding that students gave to this type of problems. Which in turn denotes in serious difficulties for teachers when facing the teaching of mixed number or improper fractions taken to the real context (Castro, 2017). According to Perera and Valdemoros (2007), they agree that one of the most complex subjects to teach in mathematics is fractions, precisely because of the different limitations that teachers have when teaching them.

On the other hand, in terms of teacher practice, teachers rarely encourage the development of metacognitive processes in their students, despite the fact that historically this type of strategy has been successful in many school environments, according to Barros et al (2011), metacognition in the classroom helps students to monitor, reflect and learn from their own ways and levels of learning, encouraging the development of awareness in students regarding how they learn and when they need help from the teacher, as well as allowing them

to set goals according to their academic process. This shows the importance of the aforementioned, deepening in the errors that students make when solving operations and problems with fractions. This tool (metacognition) is one of the most appropriate theories to be taken into account by elementary school teachers in the teaching of fractions, since it would allow them to strengthen their skills and competences by accepting and monitoring the constant errors made by their students.

Conclusions

Now, when analyzing the information collected through the techniques and instruments used in the research according to the general objective, it is concluded that the greatest presence of errors in students when operating fractions is due to difficulties in mathematical language, derived from little previous knowledge and errors associated with the rigidity of thinking that includes errors of association, interference and assimilation. Likewise, it can be evidenced that students find it difficult to argue with their own words about the concept of fraction and the elements that make it up, i.e., they do not have enough clarity and mastery to achieve a correct understanding of the subject, using knowledge inadequately. Having confusion in the notions and not assimilating the terminologies they use, such as, for example, errors in reading or writing fractions lead them to give wrong answers to the problems posed.

Taking into account the above, it is necessary to examine whether the educational proposals used as teachers are being meaningful for students and without generating any type of difficulty that leads them to make mistakes, achieving that they can understand the mathematical language in the different contents necessary for the contextualization and personalization of knowledge, in addition to carrying out activities that cause reflection on their results to recognize their faults and the need to correct them. In turn, since errors are present in students' learning, it is necessary to identify them, as well as to determine their causes and organize the way of teaching, so as to improve the teaching-learning process in students. Metacognition is proposed as a key strategy to strengthen their academic processes

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