Eco matemático ISSN: 1794-8231 (Impreso), E-ISSN: 2462-8794 (En línea) Volumen 13 (1) Enero-Junio de 2022, páginas 52-62

Forms of probabilistic reasoning of sixth grade students of secondary education

Formas de razonamiento probabilístico de estudiantes de sexto grado de educación secundaria

Arnulfo Fajardo-Valencia[a], David Benítez-Mojica[b]

[a]Doctoral student in Education, emphasis in Mathematics Education, arnulfo.fajardo@correounivalle.edu.co, https://orcid.org/0000-0002-5294-1940, Universidad del Valle, Cali, Colombia

[b]Doctor in Sciences with specialization in Educational Mathematics, david.benitez@correounivalle.edu.co, https://orcid.org/0000-0001-7421-6080, Universidad del Valle, Cali, Colombia

Forma de citar: Fajardo Valencia, A.. Benítez-Mojica, D. (2022). Forms of probabilistic reasoning of sixth grade students of secondary education. *Eco Matemático,* 13 (1), 52-62

Received: September 12, 2021
Approved: November 5, 2021

Keywords

Mathematics education,
Reasoning,
Probability,
Beliefs.

Abstract: This research seeks to identify the forms of probabilistic reasoning used by sixth-grade students when faced with situations involving random events. This is a qualitative and exploratory study based on five categories of probabilistic reasoning proposed by Sánchez and Benítez (1997), where the levels of lack of prediction, determinism, mechanical, pre-rigor and rigor determine the students' lines of reasoning when faced with problems involving probability. The data were collected from a questionnaire, semi-structured interviews and participant observation processes. Fifteen sixth-grade students of basic secondary education from an official educational institution in the city of Cali, Colombia, participated in the study. The results show that when students face problems involving the concept of probability, a level of reasoning prevails over others, and they generally resort to their beliefs to solve them, which can affect their performance in the learning processes. It is concluded that it is of great importance that teachers identify beliefs and subjective thinking elements of students before planning interventions to teach probability in classrooms.

*Autor para correspondencia: arnulfo.fajardo@correounivalle.edu.co

Doi: https://doi.org/10.22463/17948231.3301

Palabras clave

Educación Matemática, Razonamiento, Probabilidad, Creencias.

Resumen: Esta investigación busca identificar cuáles son las formas de razonamiento probabilístico utilizadas por los estudiantes de sexto grado cuando se enfrentan a situaciones que involucran eventos aleatorios. Este es un estudio de tipo cualitativo y exploratorio fundamentado en cinco categorías de razonamiento probabilístico propuestas por Sánchez y Benítez (1997), en dónde los niveles de impredicción, determinismo, mecánico, de pre-rigor y de rigor determinan líneas de razonamiento de los estudiantes al enfrentarse a problemas que involucran la probabilidad. Los datos se recolectaron a partir de un cuestionario, entrevistas semiestructuradas y procesos de observación al participante. En el estudio intervinieron 15 estudiantes de grado sexto de educación básica secundaria de una institución educativa oficial de la ciudad de Cali, Colombia. Los resultados muestran que cuando los estudiantes enfrentan problemas que involucran el concepto de probabilidad existe un nivel de razonamiento que prevalece sobre los demás, y que generalmente acuden a sus creencias, en ocasiones erróneas para resolverlos, lo que puede afectar su desempeño en los procesos de aprendizaje. Se concluye que es de gran importancia que los docentes identifiquen creencias y elementos de pensamiento subjetivo de los estudiantes antes de planear intervenciones de enseñanza en las aulas de clase.

Introduction

Previous research studies have demonstrated the importance of teaching probability even from the initial levels of school education (Gómez Torres, 2016; Inzunza, 2014; Vásquez and Alsina, 2014), considering that decision making under conditions of uncertainty is present in the daily life of individuals. Random events such as weather, diseases and games of chance, among others, are part of daily occurrences. As argued by Saldanha and Liu (2014), the main political, social, economic and scientific decisions are made using information based on probabilistic models.

This type of arguments have allowed more and more countries to include probability in their school curricula, being the United States the precursor of this educational phenomenon when the National Committee on Mathematical Requirements of the Mathematical Association of America, recommended for the first time in 1923 its study in grades 7 to 12. Subsequently, countries such as Australia, New Zealand, Spain, Singapore, Chile, among others, have made significant efforts to promote its inclusion in the curricula and in the training of teachers in the field (Vásquez and Cabrera, 2022).

Colombia has not been a stranger to the movement to include the teaching of probability in the state curriculum. In 1994, Law 115 or the General Education Law was enacted, in which the Ministry of National Education (MEN) established mathematics as a compulsory and fundamental area of the curriculum. From there on, the curricular guidelines (MEN, 1998) and the basic standards of competencies in mathematics (MEN, 2006) highlight random (probabilistic) thinking as one of the five types of thinking that are necessary to develop in order to be mathematically competent. As a result, there is a need for teachers to address probability topics to contribute to the development of random thinking in their students (Burbano-Pantoja et al., 2017) at all levels of schooling.

In Colombia as well as internationally, some researchers have focused their studies on analyzing the understanding of concepts related to chance and probability calculation (Batanero and Sanchez, 2005; Behar and Ojeda, 2016; Burbano-Pantoja et al., 2017; Ernesto Sánchez and Valdes, 2013; Watson et al., 2009). However, as recorded by Barragues and Guisasola (2009), didactic research has been pointing out that students have difficulties in achieving learning with understanding of formal concepts and procedures related to chance. Researchers agree on the importance of enhancing the development of students' probabilistic reasoning, having as a central element the understanding of the language of probability.

From this perspective, the importance of probability teaching lies in educating the probabilistic reasoning needed to cope with chance in everyday life and improve students' intuitions (Batanero, 2006), Ben-Zvi (2018) considers that probabilistic reasoning is becoming essential in society for tasks that involve dealing with uncertainty and making sense of data, Budgett and Pfannkuch (2019) state that " probabilistic reasoning is essential for operating sensibly and optimally in the XXI century" (p. 3).

Sánchez and Valdez (2015) state that this type of reasoning refers, on the one hand, to arguments that have as premises and conclusions statements of probability, meaning that they involve mathematical concepts of probability; and on the other hand, to the processes of understanding and construction of such arguments, for Batanero et al. (2016) "Probabilistic reasoning is a mode of reasoning that refers to judgments and decision making under uncertainty and is relevant to real life" (p. 9).

Research on probabilistic reasoning

The origin of research is considered on probabilistic reasoning stems from the studies of Piaget and Inhelder in 1951 (Hernandez et al., 2021; Jones and Thornton, 2005) who were interested in identifying the cognitive development processes of children and how they access probabilistic ideas with understanding. For them, only during the period of formal operations (between 11 and 15 years of age) is it possible to reason about probability, because it is there where propositional logic, proportionality and combinatorial reasoning are present.

Based on Piaget's and Inhelder's studies, other researchers made important contributions on this type of reasoning. Fischbein (1975) proved that elementary school students could establish correct ideas regarding probability concepts, as long as they were subjects of instruction; this position boosted research in didactics and curricular developments (Batanero, 2013; Groth et al., 2021). Kahneman and Tversky (1972) were concerned with inquiring about the strategies that people use to make decisions under conditions of uncertainty. The heuristics identified by Kahneman and Tversky are representativeness, availability and, adjustment and anchoring; the authors observed that these strategies can be useful for making probability estimates, but they can also generate biases or misconceptions.

Based on the work of Piaget and Inhelder (1951) and Fischbein (1975), the foundations were laid for the body of research on probabilistic reasoning, in the words of Batanero (2013) "Many researchers have subsequently tried to confirm, reject or complete the conclusions of Piaget and Fischbein". Similarly, specific aspects of the heuristics proposed by Kahneman and Tversky (1972) have been the focus of attention in much of the research on students' probabilistic learning (Jones and Thornton, 2005).

Some researchers have been interested in finding strategies, theories and cognitive models that allow observing possible transformations and patterns of improvement in students' reasoning (García and Sánchez, 2013; Jones et al., 1997; Polaki, 2002; Sánchez and Landín, 2014; Stanovich et al., 2008), these studies suggest categories or levels of growth in reasoning according to the understanding of situations related to probability.

The objective of this research is to identify which are the forms of probabilistic reasoning used by students in sixth grade of secondary education, when faced with situations involving random events.

In the development of this study, the classification proposed by Sánchez and Benítez (1997) for the analysis of students' responses is adopted. They argue that when students face problems involving probabilistic concepts, at least five levels of reasoning are evidenced, defined as follows:

1. Unpredictability level: here are located those individuals who, faced with a situation related to chance, find it completely impossible to predict an outcome.

2. Deterministic level: it groups people who explain the behavior of chance phenomena on the basis of a cause that governs them.

3. Mechanistic level: people incorporate mathematical constructs to explain chance phenomena, but they are quoted out of context, in a memorized way, giving little evidence of having understood the phenomenon.

4. Pre-rigorous level: at this level, arguments are used from a mathematical point of view, moving away from deterministic thinking, traits of combinatorial thinking are noted, however, not all possible results are obtained or more are seen than they really are.

5. Level of rigor: here are located the people who argue mathematically, the use of different representations to explain the behavior of the event (tables, graphs, equations) is observed. The arguments used are correct from the mathematical point of view.

Method

In this research, an exploratory study was conducted under a qualitative research approach. As Hernández et al. (2010) point out, "exploratory studies are conducted when the objective is to examine a topic or research problem that has been little studied, about which there are many doubts or which has not been addressed before" (p. 79), in this case we seek to obtain information from the responses of a group of 15 students in sixth grade of basic secondary education in an official educational institution in the city of Cali, Colombia, in order to identify which are the forms of probabilistic reasoning used when faced with situations involving random events.

The proposed activities were carried out during regular mathematics classes in which 9 females and 6 males participated, all belonging to an age range between 11 and 14 years old.

The techniques used to collect the information were the survey and the interview, in this study the interview is assumed as a research strategy that seeks "to collect spontaneous and authentic ideas from people or groups of people, opinions or points of view to analyze from them reasoning processes or conceptions and even detect thought structures that underlie the ideas exposed" (Camargo, 2021, p. 73). The instruments used for data collection were a questionnaire and, in the case of the interview, a semi-structured questionnaire.

The initial questionnaire consisted of eight multiple-choice questions, three of which were related to the sample space and the probability of an event and five to probability comparisons, and was applied directly to the students by the teacher responsible for the subject in the presence of the researcher. To carry out the activity, a time of 60 minutes was assigned; students were asked to pay special attention to the written justification of each of the selected answers.

The students were interviewed individually by the researcher, based on the answers to the questions and the justifications given in the initial questionnaire.

Data analysis

Table I presents the percentages of correct and incorrect answers given by the students from the normative point of view of probability.

Table I. Correct and incorrect answers from the normative point of view of probability.

	Question 1	Question 2	Question 3	Question 4	Question 5	Question 6	Question 7	Question 8
CA	20%	67%	47%	7%	53%	33%	53%	7%
WA	80%	23%	53%	93%	47%	67%	47%	93%

CA: correct answer ,WA: wrong answer

Below are some examples of the way in which the students' answers were classified based on the levels of probabilistic reasoning proposed by Sánchez and Benítez (1997) and some forms of reasoning of the students are evidenced from the justifications given to the answers of the questionnaire, as well as in the interviews carried out for the study.

In the first item of the questionnaire, students were shown the image of a bag containing 5 white balls and 8 black balls and were asked:

If we take randomly and without looking at a ball from the bag, what is most likely to happen?

a. Take out a white ball
b. Take out a black ball
c. Both of the above events are equally likely
d. The outcome cannot be predicted

Some of the students' responses and justifications (E) were:

E5: Response d. Because when I take out the ball I couldn't tell if I took out the white ball or the black ball so you can't predict.

E6: Answer c. Because when you put your hand in and start to move it, the balls start to move and either color ball could be pulled out.

E7: Answer c. Because we could put our hand in the back or not and in conclusion either one could be taken out.

According to the categories established by Sánchez and Benítez (1997), this type of arguments place students in the *Imprediction Level*. It can be observed in their comments the difficulty to predict the result of the probabilistic event, they do not establish relations between the number of elements and much less between sets, there is no coherence between their appreciations and the type of answer selected.

Item 2 of the questionnaire poses the following question:

You toss a two hundred pesos coin three times in a row, which of the following combinations of heads and seal, occurs with the highest probability?

a. Two heads and tails
b. Three heads.
c. Three tails.
d. All of the above events have the same probability of occurrence.

Some of the justifications for the students' responses were:

E2: Response d. There may be something for it to fall one of the two ways, such as the way it is tossed, the wind alone, the tilt etc. but it all depends on your luck.

E5: Answer d. You can't tell how many times it will land on each side, because if the coin is flipped too hard we wouldn't know what came out.

E8: Answer d. It is already a question of the force of gravity.

This type of students' arguments are clearly established in the Deterministic Level, there is a tendency to interpret phenomena related to randomness from possible causes, such as physical properties of objects, beliefs such as luck; the power of God; destiny, influenced the way in which the individuals surveyed faced situations related to probabilistic concepts.

Some excerpts from interviews also made it possible to evidence the inclination of students to deterministic justifications. As an example, a segment of the dialogue between the interviewer and one of the students in the group is presented:

Interviewer: if you were given a choice between numbers 111, 379 and 234 to participate in a raffle, which one would you choose and why?

Student: 379, because my lucky number is 37.
Interviewer: Have you ever won a drawing with your lucky number?

Student: well I don't usually bet, but when you pick a number for a raffle, you think of a lucky number, or the date of your birthday. For example, my cousin's favorite number is 66.

In some answers, traits in the level of mechanical reasoning of the students are identified when they present arguments associated with terms from the language of probability, but disjointed from the phenomenon being analyzed. Examples of this type of reasoning were established from the justifications given to the following question:

Suppose you participate in a lottery, in which you have the number 1234.

Which is more likely?

a. That all four digits will come out.
b. That the last three digits will be drawn.
c. Let the last two digits be the last two digits.
d. All of the above events have the same probability of occurring.

Some of the student responses were:

E2: Answer C. They might come out since the in most of the times the 3 or 4 are there, even so, it is a probability of ¼ each.

E11: Answer D. It is something random, you can't know, in everything there is the same probability.

E14: Answer D. They have the same probability because they don't tell me any significant data that I can use to say that it is more likely.

The use of terms that involve probabilistic concepts can be observed, even establishing relationships of proportionality, which, however, do not lead the student to give a normatively correct answer.

In question 5, students were presented with two containers with black and white balls, box one contained 6 black balls and 8 white balls, while box two contained 3 black balls and 4 white balls, students were asked: Without seeing inside the boxes, which is more likely, to get a black ball from box one or from box two?

Some of the students' responses were:

E8: in both boxes there is the same probability, because there are more white balls than black balls.

E13: I think it is the same probability since in either case there are few black balls compared to white balls.

This type of justifications for this grade of formal education can be considered at the pre-rigor level, since students establish proportionality relationships, identify the number of favorable cases from comparisons between the elements that are part of the random event, although they do not refer to the sample space, nor do they establish a numerical representation for probability.

Likewise, during the development of the study, it was not observed that the students used different types of representation to those offered in the questionnaire, they did not use tables, graphs or drawings in the search for solutions to the questions, few mathematical resources were used, so it was not considered during the analysis of the data that they did not reach levels of rigor in their reasoning.

Table II shows the consolidated results of the analysis of the justifications given by the students to their answers in the questionnaire, according to the levels of reasoning proposed by Sánchez and Benítez (1997).

Table II. Percentage of students' justifications by level of probabilistic reasoning.

Impredictión Level	Deterministic Level	Mechanistic level	Pre-rigorous level	Level of rigor
54,16%	22,50%	11,67%	11,67%	0%

It can be observed that 76.66% of the students' answers are located in the levels of unprediction and deterministic reasoning, while in the mechanical level and in the pre-rigor level the percentage is only 11.6%, none of the justifications given by the students had a level of probabilistic rigor according to the established categories.

Table III shows the percentages obtained from the justifications given by the students for each of the questions in the questionnaire.

Table III. Percentage per question according to the levels of probabilistic reasoning.

Question	Impredictión	Deterministic	Mechanistic	Pre-rigorous	Rigor
1	53,3%	20,0%	0%	26,6%	0%
2	26,6%	53,3%	6,6%	13,3%	0%
3	46,6%	0%	33.3%	20,0%	0%
4	73,3%	6,6%	20,0%	0%	0%
5	46,6%	26,6%	6,6%	20,0%	0%
6	66,6%	20,0%	6,6%	6,6%	0%
7	53,3%	26,6%	20,0%	0%	0%
8	66,6%	26,6%	0%	6,6%	0%

As in Table II, the levels of unpredictability and determinism are the highest, the approximations to levels of pre-rigor and rigor are scarce, the students present difficulties to differentiate between random and deterministic events, they do not use mathematical arguments, such as proportionality relations between favorable and unfavorable cases, in very few cases they identify the sample space of a random experiment and they do not apply Laplace's rule to obtain probabilities.

Conclusions

When students face questions involving the concept of probability, it is observed that there is a line of reasoning that prevails over the others; in this particular study, the participants are at levels of unprediction and determinism, with subjective and causal thinking prevailing in these cases.

The results of the research coincide with previous studies such as those conducted by Watson et al. (2004) and Sharma (2006) where students associate random events with ideas such as luck, fate, divine intervention. When asked about events associated with random generators such as dice or coins, students base their reasoning on physical phenomena such as the influence of wind, the weight of bodies and human control over objects, these ideas had also been previous findings in the works of Ang and Shahrill (2014) and Nicolson (2005).

As in the aforementioned research, Colombian students, rarely used mathematical resources when faced with problems involving random phenomena. In most cases, they resorted to their beliefs resulting from previous experiences and the influence of their sociocultural environment. The idea is shared that these erroneous beliefs affect their performance in learning processes with an understanding of probability.

The coincidence among the results of this study carried out in Colombia and the results reported in the research literature in other parts of the world show that, just as the study of probability should always be present in school curricula, it is also necessary to strengthen the didactic training of teachers in this field and to create bridges between research in probability didactics and teaching practices in the classroom.

From the analysis of the information collected in the study, it is recommended that teachers carry out activities that allow identifying the beliefs that students have associated with chance and randomness; this can facilitate the understanding of their ways of reasoning and can result in the improvement of the planning of the probability teaching processes that are developed in the classroom. As Fajardo and Benítez (2020) state, knowledge of the belief system constitutes the baseline from which the teacher starts to design tasks.

References

Ang, L. H., & Shahrill, M. (2014). Identifying Students' Specific Misconceptions in Learning Probability. *International Journal of Probability and Statistics, 3*(2), 23–29. https://doi.org/10.5923/j.ijps.20140302.01

Barragues y Guisasola. (2009). Una propuesta para la enseñanza de la probabilidad en la universidad basada en la investigación didáctica. *Educación Matemática, 21*(3), 127–162. Recuperado de: http://funes.uniandes.edu.co/13192/1/Barragues2009Una.pdf

Batanero, C. (2013). (2013). La comprensión de la probabilidad en los niños. ¿Qué podemos aprender de la investigación? En J. A. Fernandes, P. F. Correia, M. H. Martinho, & F. Viseu,. *Atas Do III Encontro de Probabilidades e Estatística En La Escuela. Braga: Centro de Investigación En Educación. Universidad Do Minho*, 1–13. Recuperado de: https://www.ugr.es/~batanero/pages/ARTICULOS/1Batanero.pdf

Batanero, Carmen. (2006). Razonamiento probabilístico en la vida cotidiana: un desafío educativo. *Investigación En El Aula de Matemáticas. Estadística y Azar*. http://www.ugr.es/~batanero/pages/ARTICULOS/ConferenciaThales2006.pdf

Batanero, Carmen, Chernoff, E. J., Engel, J., Lee, H. S., & Sánchez, E. (2016). *Research on Teaching and Learning Probability*. https://doi.org/10.1007/978-3-319-31625-3_1

Batanero, Carmen, & Sanchez, E. (2005). What is the Nature of High School Students' Conceptions and Misconceptions About Probability? *Exploring Probability in School*, 241–266. https://doi.org/10.1007/0-387-24530-8_11

Gutierrez, R., & Ojeda, M. M. (2016). El papel de algunas opciones tecnológicas en la educación estadística. *Heurística*, 18, 21–35. Recuperado de: https://www.researchgate.net/profile/Mario-Ojeda/publication/312398845_EL_PAPEL_DE_ALGUNAS_OPCIONES_TECNOLOGICAS_EN_LA_EDUCACION_ESTADISTICA/links/587d07c808ae9a860ff0e472/EL-PAPEL-DE-ALGUNAS-OPCIONES-TECNOLOGICAS-EN-LA-EDUCACION-ESTADISTICA.pdf

Ben-Zvi, D. (2018). Foreword. In & E. P. A. Leavy, M. Meletiou-Mavrotheris (Ed.), *Statistics in early childhood and primary education: supporting early statistical and probabilistic thinking* (pp. vii–viii). Springer Singapur. Recuperado de: https://www.researchgate.net/publication/326587840_Preface_to_volume_Statistics_in_Early_Childhood_and_Primary_Education_Supporting_Early_Statistical_and_Probabilistic_Thinking

Budgett, S., & Pfannkuch, M. (2019). Visualizing Chance: Tackling Conditional Probability Misconceptions. In G. Burrill & Ben-Zvi (Eds.), *Topics and Trends in Current Statistics Education Research, Monographs ICME-13* (pp. 3–25). Springer International Publishing. https://doi.org/10.1007/978-3-030-03472-6_1

Burbano-Pantoja, V. M. Á., Valdivieso-Miranda, M. A.,&Aldana-Bermúdez,E.(2017).Conocimiento base para la enseñanza: un marco aplicable en la didáctica de la probabilidad. *Revista de Investigación, Desarrollo e Innovación, 7*(2), 269–285. https://doi.org/10.19053/20278306.v7.n2.2017.6070

Camargo, L. (2021). *Estrategias Cualitativas de investigación en educación matemática* (1st ed.). Ed. Universidad de Antioquia-Universidad Pedagógica Nacional de Colombia. Recuperado de: https://conferencia.ciaem-redumate.org/index.php/xvciaem/xv/paper/viewFile/1061/568

Fajardo & Benítez, D. (2020). influencia de las creencias de los estudiantes en la resolución de problemas en educación matemática. *Revista de Educación Matemática, 35*(3). Recuperado de: https://revistas.unc.edu.ar/index.php/REM/article/view/28106

Fischbein, E. (1975). *The intuitive sources of probabilistic thinking in children*. Recuperado de: https://link.springer.com/book/10.1007/978-94-010-1858-6

García, J., & Sánchez, E. (2013). Niveles de razonamiento probabilístico de estudiantes de bachillerato frente a una situación básica de variable aleatoria y distribución. *Actas de Las Jornadas Virtuales de Didáctica de La Estadística, Probabilidad y Combinatoria* , 417–424. Recuperado de: https://www.researchgate.net/publication/326158345_Niveles_de_razonamiento_probabilistico_de_estudiantes_de_bachillerato_frente_a_una_situacion_basica_de_variable_aleatoria_y_distribucion

Gómez Torres, E. (2016). Estadística Y Probabilidad En El Currículo Colombiano Para Educación Básica Y Media. *XXVI Simposio Internacional de Estadística 2016*, 1–5. Recuperado de: https://docplayer.es/42683957-Estadistica-y-probabilidad-en-el-curriculo-colombiano-para-educacion-basica-y-media.html

Groth, R. E., Austin, J. W., Naumann, M., & Rickards, M. (2021). Toward a theoretical structure to characterize early probabilistic thinking.

Mathematics Education Research Journal, 33(2), 241–261. https://doi.org/10.1007/s13394-019-00287-w

Hernandez, A., Batanero, C., Gea, M., & Rocio, A. (2021). *Research on children' s reasoning in comparing probabilities probabilities. March*. Recuperado de: https://www.researchgate.net/publication/350103195_Research_on_children's_reasoning_in_comparing_probabilities

Hernández, R., Fernández, C., & Baptista, L. (2010). *Metodología de la Investigación* (5th ed.). México D.F. McGraw-Hil. Recuperado de: https://www.icmujeres.gob.mx/wp-content/uploads/2020/05/Sampieri.Met.Inv.pdf

Inzunza, S. (2014). Geogebra: Una herramienta cognitiva para la enseñanza de la probabilidad. *Congreso Iberoamericano de Ciencia, Tecnología, Innovación y Educación*, 1–11. Recuperado de: https://docplayer.es/38490613-Geogebra-una-herramienta-cognitiva-para-la-ensenanza-de-la-probabilidad.html

Jones, G. A., Langrall, C. W., Thornton, C. A., & Timothy Mogill, A. (1997). A framework for assessing and nurturing young chiedren's thinking in probabieity. *Educational Studies in Mathematics*, 32(2), 101–125. https://doi.org/10.1023/A:1002981520728

Jones, G. A., & Thornton, C. A. (2005). An Overview of Research into the Teaching and Learning of Probability. In *Exploring Probability in School* (pp. 65–92). https://doi.org/10.1007/0-387-24530-8_4

Kahneman, D., & Tversky, A. (1972). Subjective probability: A judgment of representativeness. *Cognitive Psychology*, 3(3), 430–454. https://doi.org/10.1016/0010-0285(72)90016-3

MEN. (1998). Lineamientos Curriculares de Matemáticas. Cooperativa Editorial Magisterio, 103. Recuperado de: https://www.studocu.com/co/document/universidad-del-valle-colombia/matematica-basica/lineamientos-curriculares/9530727

MEN. (2006). Estándares Básicos de Competencias en Matemáticas. *Estándares Básicos de Competencias En Lneguaje, Matemáticas, Ciencias y Cuidadanas*, 46–95. https://doi.org/958-691-290-6

Nicolson, C. (2005). Is chance fair? A student's thoughts on probability. *Teach Kids Math*, 12(2), 83–89. Recuperado de: https://eric.ed.gov/?id=EJ756749

Piaget, J., & Inhelder, B. (1951). *La genèse de l'idée de hasard chez l'enfant*.Recuperado de: https://issuu.com/ediciones_morata/docs/piaget_psicolog__a_del_ni__o/65

Polaki, M. V. (2002). Using instruction to identify mathematical practices associated with Basotho elementary students' growth in probabilistic thinking. *Canadian Journal of Science, Mathematics and Technology Education*, 2(3), 357–370. https://doi.org/10.1080/14926150209556526

Saldanha, L., & Liu, Y. (2014). *Challenges of Developing Coherent Probabilistic Reasoning: Rethinking Randomness and Probability from a Stochastic Perspective*. 367–396. https://doi.org/10.1007/978-94-007-7155-0_20

Sánchez & Benítez, D. (1997). Algunos acercamientos al pensamiento probabilista de los alumnos. *Actas de La Undécima Reunión de Matemática Educativa. Relme*, 157–161.

Sánchez, E, & Valdez, J. (2015). El razonamiento probabilístico informal de estudiantes de bachillerato. In C. Fernandez, M. Molina, & N. Planas (Eds.), *Investigación en Educación Matemática XIX* (pp. 89–103). Recuperado de: http://funes.uniandes.edu.co/8769/1/Sanchez2015Razonamiento.pdf

Sánchez, Ernesto, & Landín, P. R. (2014). *Levels of Probabilistic Reasoning of High School Students About Binomial Problems*. 581–597. https://doi.org/10.1007/978-94-007-7155-0_31

Sánchez, Ernesto, & Valdes, J. (2013). La cuantificación del azar: Una articulación de las definiciones subjetiva, frecuencial y clásica de probabilidad. Probabilidad Condicionada: *Revista de Didáctica de La Estadística*, ISSN-e 2255-5854, No. 1, 2013, Págs. 39-46, 1, 39–46. Recuperado de: https://www.ugr.es/~jmcontreras/pages/Investigacion/Actas%20jornadas.pdf

Sharma, S. (2006). *Personal Experiences and Beliefs in Early Probabilistic Reasoning: Implications for Research. 1*(1), 177–184. Recuperado de: https://files.eric.ed.gov/fulltext/ED496961.pdf

Stanovich, K. E., Toplak, M. E., & West, R. F. (2008). The development of rational thought: A taxonomy of heuristics and biases. In *Advances in Child Development and Behavior* (Vol. 36). Elsevier B.V. https://doi.org/10.1016/S0065-2407(08)00006-2

Vásquez, C., & Alsina, Á. (2014). Enseñanza de la probabilidad en educación primaria. Un desafío para la formación inicial y continua del profesorado. Números. *Revista de Didáctica de Las Matemáticas*, 85, 5–23. Recuperado de: http://funes.uniandes.edu.co/3677/1/V%C3%A1squez2014Ense%C3%B1anzaNumeros85.pdf

Vásquez, C., & Cabrera, G. (2022). La estadística y la probabilidad en los currículos de matemáticas de educación infantil y primaria de seis países representativos en el campo. *Educación Matemática*, 34(2), 245–274. https://doi.org/10.24844/em3402.09

Watson, J., Caney, A., & Kelly, B. (2009). Development of student understanding of outcomes involving two or more dice. *International Journal of Science and Mathematics Education, 7*, 25–54. Recuperado de: https://www.semanticscholar.org/paper/Development-of-Student-Understanding-of-Outcomes-or-Watson-Kelly/396c34589d5a9154922449238d34dd0cb7c45f29

Watson, J. M., Caney, A., & Kelly, B. A. (2004). Beliefs about Chance in the Middle Years : Longitudinal Change. *In Mathematics Education for the Third Millennium: Towards 2010. Proceedings of the 27th Annual Conference of the Mathematics Education Research Group of Australasia*, 581–588. Recuperado de: https://www.academia.edu/32818867/Beliefs_about_Chance_in_the_Middle_Years_Longitudinal_Change.