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A short heuristic-mathematical Model of Social Mobility based in Kermack-McKendrick hypothesis.

Un breve modelo heurístico-matemático de movilidad social basado en la hipótesis de Kermack-McKendrick.

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Palabras clave

Movilidad Social, Modelo, Heurística, Distribución Del Ingreso, Hipótesis De Kermack-Mc Kendrick.

Resumen: El siguiente manuscrito analiza la relación existente entre la movilidad social y un modelo epidemiológico, desde un enfoque heurístico. El modelo epidemiológico (hipótesis de Kermack-Mc Kendrick), puede constituirse en una herramienta de análisis y de mejora de condiciones y aspectos relacionados con la distribución del ingreso y como herramienta para el avance en la movilidad social. El texto considera que las posibilidades del modelo pueden contribuir al corpus teórico de otras disciplinas como la ciencia económica y principalmente al estudio de los factores que afectan la movilidad social y que generalmente se asocian con variables como el nivel de educación. De igual manera, se apoya en la heurística como elemento esencial para la investigación de nuevos caminos de análisis de la realidad económica y en específico de su relación con la movilidad social.

Keywords

Social Mobility, Model, Heuristic, Income Distribution, KMH.

Abstract: The following manuscript analyzes the relationship between social mobility, heuristics, and the proposal of a model of epidemiological origin (Kermack-McKendrick Hypothesis -KMH), which can become a tool for the analysis of conditions improve aspects related to income distribution as a fundamental tool for the advancement of social mobility. The text considers that the proposed model considers the possibilities embodied by the contribution of other disciplines to economic science and mainly concerning the study of the factors that affect social mobility, generally associated with variables such as the level of education. Likewise, heuristics constitute an essential tool to search for new ways of analyzing the economic reality, as far as social mobility is concerned.

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Introduction

All modern societies have been characterized by a state of stratification, wherein one way or another there has been a social hierarchy based on wealth or status. Although this classification has harmed society and has generated a new debate on the need for social correctives that enact equality, recognizing the negative impact of race, wealth, and even caste in some societies on social mobility, it should not be overlooked that social mobility is also associated with the rationality of economic agents. Are the principles of the rationality of economic agents justifiable as an explanation for the problems of income distribution and social mobility? Stratification should be seen as a resource for public policy action and, in general, as a mechanism to advance redistribution and an a *priori* condition for better decision making, but by itself it ensures nothing.

However, in many cases, the stratification mechanism generates segregation and discrimination, which are accompanied by slow action by the economic authorities to ensure an equitable distribution of national wealth and the consolidation and effectiveness of models that guarantee social mobility and positively affect the different social strata.

Stratification at the social level, then, emerges as a concept that derives in a complex way in political and mainly economic dimensions. In some social systems, whose rigidity does not allow a framework of action for state policies, mobility tends to be less plausible. It could even be argued that social choices are undoubtedly defined by a greater scope for social mobility, ranging from individual resource allocation to future expectations. Unfortunately, this influences how hierarchization generates an unjust situation and becomes a breeding ground for violence.

A clear example that may allow us to understand how stratification generates the impossibility of social ascent is related to the way and conditions in which an individual can move in the social queue, according to certain contextual characteristics that will define the flexibility of the hierarchy. But individual movement up the social ladder is not indicative of how a social group ascends or positions itself in strata with greater possibilities of development and well-being *"one swallow does not make a summer".*

Faced with this situation of an erroneously naturalized order, the social disciplines must generate strategies that allow for the correction of social inequalities, the basis of which is precisely access to adequate income, to fundamental services, and, above all, to education as a key device in the generation of these differences. On this depends, then, that economic science overcomes formal aspects of the presentation of theories and models and that these can be applied to a reality that demands it. It is not in vain that classic measurements such as the Gini coefficient show areas, especially in the African continent and Latin America, as areas where there is widespread inequality.

The document is organized in the first part focused on the Gini coefficient and some indicators on income redistribution, followed by a brief analysis of the relationship between development and investment. The second part presents elements related to heuristic models in economics. In the third part, a formal presentation of the model is made and finally, some discussion points are proposed regarding the scope of the model.

Theoretical Background

Gini Coefficient According to Farris (2010), the Gini index is calculated based on the Lorenz curve and can be characterized as the integral that defines the area concerning the line of equality. The consideration of the existence of a whole model that

graphically shows the explanation through geometric methodologies, of the calculation of areas, corresponds without a doubt to a heuristic procedure with rules.

$$G := 2\int_0^1 [p - L(p)]dp \qquad (1)$$

Where L refers to the Lorenz curve. The following figure shows the existing differences in terms of inequality that are analyzed from a box that presents a diagonal with a positive slope in whose points are located those desirable scenarios such as perfect distribution in which case G1 = 0. As the Lorenz curve distances itself from the line of equality, new scenarios appear such as G2 > 0, called unequal scenarios. This situation can be much more marked, and the Lorenz curve can be much more unequal, as in scenario 3, where the Lorenz curve is steeper G2 > 0. And finally, we can have an unequal scenario as shown in graph 4, where the value of the Gini coefficient G4 = 1.

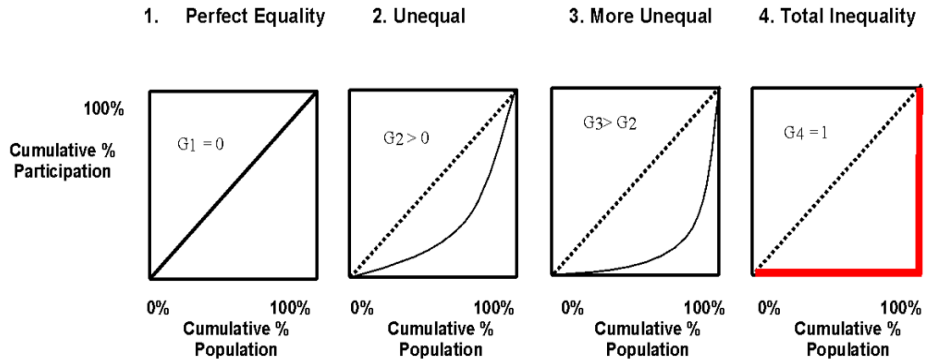


Figure 1. Different scenarios in the measurement of the Gini coefficient.

Source: Own construction.

From a total of 156 countries, the World Bank provides the fundamental statistics to analyze the concentration of wealth at a global level. Thus, we find the following summary table of statistics showing aspects related to this level of concentration. The global Gini coefficient - considering, however, that the measurements are taken in different years - is 38.54872 with a maximum level of 63 for South Africa and a value of 25 for Ukraine . Below are the associated statistical data:

Table I. Summary Statics Gini Coefficient.

Variable	Obs	Mean	Std. Dev.	Min	Max Std. Err.	[95% Conf.	Interval]
Gini	156	38.54872	7.867184	25	63 .6298788	37.30446	39.79297

Source: own construction base on World Bank, Development Research Group.

According to World Bank data, the first data are generally from countries in these geographic areas mentioned above:

Table II. Gini coefficient 20th positions.

Position	Countries	Gini	Year
1	South Africa	63	2014
2	Namibia	59.1	2015

3	Suriname	57.6	1999
4	Zambia	57.1	2015
5	Central African Republic	56.2	2008
6	Lesotho	54.2	2010
7	Mozambique	54	2014
8	Belize	53.3	1999
9	Brazil	53.3	2017
10	Botswana	53.3	2015
11	Swaziland	51.5	2009
12	St. Lucia	51.2	2016
13	Guinea-Bissau	50.7	2010
14	Honduras	50.5	2017
15	Panama	49.9	2017
16	Colombia	49.7	2017
17	Congo Republic of	48.9	2011
18	Paraguay	48.8	2017
19	Mexico	48.3	2016
20	Guatemala	48.3	2014

Source: own construction base on World Bank, Development Research Group.

South Africa is one of the most inequitable countries in the world, compared to other Latin American countries such as Brazil, Honduras, Panama, Colombia, Paraguay, Mexico, and Guatemala, which shows a negative influence of the generation of wealth in these areas of the world with a wide concentration. Although this concentration has a negative influence on the progress of the different countries in the consolidation of their social policies, the strategies for dealing with these realities are undoubtedly necessary justification to show how new heuristic models can deepen the analysis of the situation and especially how countries measure internal changes in the distribution of income and how these moves following the prescriptions of social policy. The following map shows the Gini coefficient by country:

Figure 2. Map of Gini coefficient.

Source: World Bank (2019).

The following table shows the result of the Gini coefficient for 21 countries with the highest levels of inequality in the years 2017-2019. According to World Bank statistics, high levels of concentration and inequality are found in countries such as Brazil, Colombia, Panama, and it is worth noting that Latin American countries unfortunately present negative statistics of line of equality:

Table III. Gini coefficient 2018-2020.

Country Name	2017	2018	2019
Brazil	53.3	53.9	53.4
Colombia	49.7	50.4	51.3
Zimbabwe	44.3	.	50.3
Panama	49.9	49.2	49.8
Honduras	49.4	48.9	48.2
Costa Rica	48.3	48	48.2
Paraguay	48.5	46	45.7
Ecuador	44.7	45.4	45.7
Argentina	41.1	41.3	42.9
Dominican Republic	42.2	43.7	41.9
Turkey	41.4	41.9	41.9
Bolivia	44.6	42.6	41.6
Peru	43.3	42.4	41.5
Uruguay	39.5	39.7	39.7
El Salvador	38	38.6	38.8
Indonesia	38.1	37.8	38.2
Georgia	37.9	36.4	35.9
Thailand	36.5	36.4	34.9
Armenia	33.6	34.4	29.9
Kyrgyzstan	27.3	27.7	29.7
Ukraine	26	26.1	26.6

Source: own construction based on Bank World Development Research Group.

The figure shows the behavior of the Gini´s coefficient for 21 countries.

Inequality, as measured by the Gini coefficient, has behaved similarly in the 2018-2020 time, for the first 21 countries in the above list and has been concentrated at Gini´s close to almost 50 in all three years. The Gini coefficient, however, is only one of the measures that allow us to analyze changes in the distribution of countries, but it has proven to be one of the most important in its contribution to development studies.

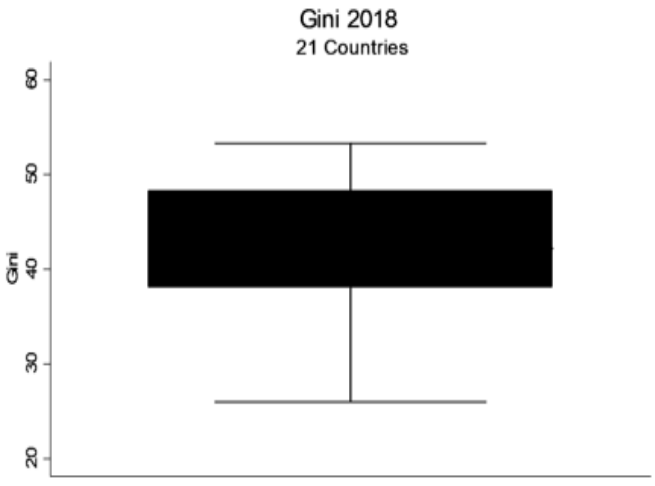
On the other hand, the coefficient is an indicator for making decisions regarding public policy on income redistribution and a first glimpse for deducing movements in the social strata, in terms of the options of social groups to obtain better incomes that will allow them to improve their living conditions.

In this sense, Karol, Nielek and Wierzbick (2014) state that there is a multi-agent system that allows demonstrating that social mobility is related to i) the idea that egalitarian societies tend to be more efficient in the management of their resources and their equitable distribution; ii) advanced procedures in the allocation of resources do not always have an impact on the restructuring of stratification, and even "perpetuate" it; iii) privileged agents, as rational actors, will prefer or see stratification as more beneficial. Stratification was historically based on the differentiation between privileged and non- privileged, without this necessarily being binary, as could be analyzed in the class groups in early industrial society. The existence of different hierarchical structures makes the analysis of social mobility, which is still based on the Weberian triad of wealth, prestige, and power, more complex.

Although the second premise, which assumes greater efficiency of egalitarian societies, reducing aspects such as crime and infant mortality, has irrefutable empirical evidence, there are authors such as Rowlingson (2011) who consider that there is no correlation between high income and social problems. On the other hand, Atkinson (1997) himself dares to affirm that economic inequality is the basis for economic growth.

Some antecedents in the study of social mobility can be found in Payne (1989), whose concept became preeminent in British sociology in the 1970s. Even authors such as Goldthorpe (Goldthorpe et al. 1980; 1987), recognize aspects related to the number of projects derived from this notion and the levels of collaboration in studies with data analysis thanks to international cooperation. The great turning point came in the 1970s with the organization of the Oxford Social Mobility Group, which is a preponderant group in the development of the concept of social mobility (Payne 1989).

Some authors consider that the unit of analysis of the mobility process should not be social class but socio-economic or occupational status (Marqués 2015). On the other hand, there is fundamental evidence that mobility is an element that generates feelings in the face of one's own voice, a voice that also generates a sense of confidence and loyalty to the State (OECD 2018, 25).



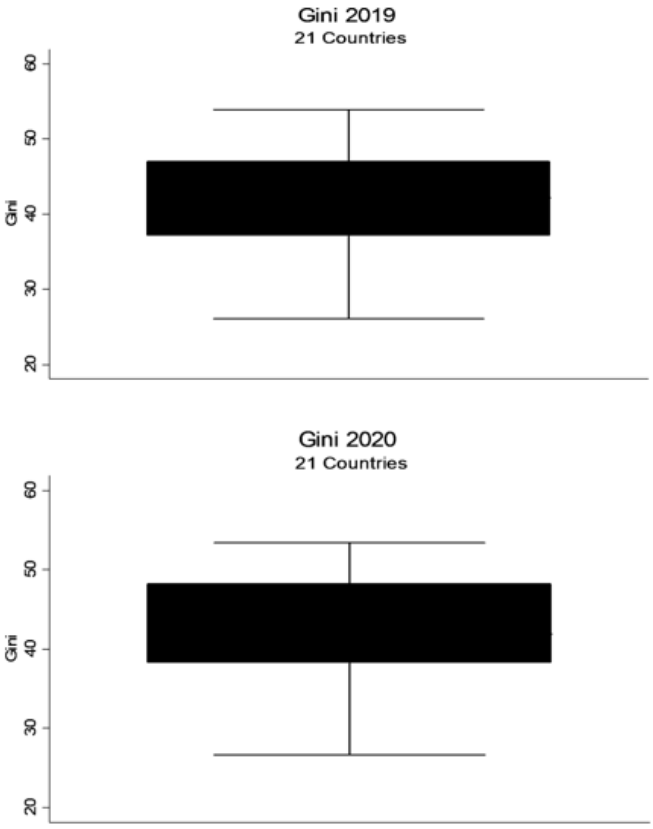


Figure 3. Box-plot Gini coefficient 2018-2020.

Source: Own construction based on World Bank, Development Research Group.

Traditionally, traditional liberal theory has been analyzed as being centered on education, technology, and good pay for work as important elements for understanding the process of social mobility or immobility, with an emphasis on education. But the educational level has not really been an important process, since it does not know -positional characteristics-, as the positional status the income (Garay 2018, 17).

At this point, it becomes necessary to be able to study the heuristic models that provide a frame of reference to find new horizons in the analysis of redistribution and mobility problems, as well as of a model that starting from the theoretical assumptions of Dongmei et al (2014), as well as some comparisons about countries in their redistribution and mobility situation. Finally, the paper raises some final considerations regarding the scope of its application in different contexts.

But not only measurements of income redistribution focus specifically on the Gini coefficient. On the other hand, we have a whole series of indicators and indexes that attempt to measure the concentration of wealth and especially how social strata access resources through state policies. Some of these indicators are consolidated below:

Table IV. Mainly measures of income distribution.

Measure	Definition
Income share of the richest *R%*	$S_R = \frac{\sum_{i=n}^{n^{100}}(1-R)+1^{x_i}}{nu_{..}}$
Income share of the poorest *P%*	$S_{P=\frac{\sum_{i=1}^{n^P} x_i}{n\mu}}$
90-10 ratio	$R_{90-10} = \frac{x_n 90}{x_{..} 10}$
R% - P% ratio	$R_{R-P} = \frac{x_n R}{x_{..} P}$
Gini	$G = \frac{-(n+1)}{n} + \frac{2}{n^2 \mu_x} \sum_{i-1}^{n} i x_i$
Thiel's first measure	$T=\frac{1}{n}\sum_{i=1}^{n} \frac{x_i}{\mu_{..}} \ln\left(\frac{x_i}{\mu_{..}}\right)$
Atkinson	$A_\varepsilon = \begin{cases} 1-\left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i}{\mu_x}\right)\right]^{\frac{1}{\varepsilon}} \; for\; \varepsilon \geq 1 \; and \; \varepsilon \neq 0 \\ 1-\prod_{.}^{n}\left(\frac{x_i}{\mu_x}\right)^{\frac{1}{n}} \; for\; \varepsilon \neq 0 \end{cases}$
Generalized Entropy	$I_\alpha = \begin{cases} \frac{1}{\alpha(1-\alpha)}\frac{1}{n}\sum_{i-1}^{n}\left[1-\left(\frac{x_1}{\mu_x}\right)^{\alpha}\right] for\; \alpha \neq 0,1 \\ \frac{1}{n}\sum_{i-1}^{n}\frac{x_i}{\mu_x}\ln\left(\frac{x_i}{\mu_x}\right) for\; \alpha = 1 \\ \frac{1}{n}\sum_{i-1}^{n}\ln\left(\frac{\mu_x}{x_i}\right) for\; \alpha = 0 \end{cases}$

Source: Fields (2001).

Where recipients are ordered from lowers income o highest income and x_i is the income of recipient *i*; μ_x is the average income and n total number of recipients.

These redistribution measures are linked to how states analyze the internal distribution situation, and the establishment of distributive policies is presented under the so-called development dilemma which, in terms of Patman (1983), encompasses the following elements:

Table V. The Development Dilemma

Sources of development Finance	Gross investment as a percentage of national income	Allocation of investment by sectors	Capital output ratios by sectors.	Increase in national output by sectors.	Increase in national output and real wealth.	Increase in per capita real incomes with a Population growth of 2.6%
Domestic Savings 17,5%	20%	Agriculture 20%	1:1	4%		

Foreign Aid 2,5%		Industry 35%	1:4	1.75%	6.95%	2,7%
		Power and transport 10%	1:7	1%		
		Social 10%	1:10	0,2%		

Source: Patman (1983).

Based on the ideas of W.W. Rostow, since the 1950s, the progressive perspective of economic growth has been recognized. Rostow, since the fifties, the progressive perspective of economic growth is recognized, based on observations made in countries such as Libya, Pakistan, and the positive perspective for Mexico, Brazil, Argentina, and Turkey. There is even a mention of the case of Colombia, where several studies of the United Nations have emphasized the economic and technical progress introduced by the coffee spread in the 1860s (Patman 1983, 49).

Heuristic models

Beyond the pioneering work of Polya (1962, 1964), heuristic models can be applied to economic analysis, considering several fundamental elements. For their application, one must consider the existence of auxiliary means, principles, rules, strategies, and programs that are focused on the resolution of practical problems of the economy, such as, for example, income distribution, a complex problem from the point of view of economic policy. When algorithmic procedures for solving various problems are not available, the auxiliary means provided by heuristics can generate a significant change in the way in which alternatives are found for new formulations to redistribute income that promotes social mobility.

Horst Müller (1987, 2015) points out that heuristic methods are ways of working and thinking that accompany the performance of demanding mental activities. In this sense, heuristic procedures can be divided into principles, rules, and strategies:

Table VI. Characteristics of heuristic models.

Principles	Rules	Strategies
They constitute suggestions that allow the direct creation of a solution idea. Their formulation allows a contribution about the means to be used, as well as the proposed solution. Among these principles, we can observe, for example, analogy, reduction, and/or modeling, among others.	The rules act as impulses that allow finding the means that lead to a possible solution. Some of the recommendations regarding the rules are found in the differentiation between what is given and what is sought, in the realization of figures of analysis such as diagrams, tables and/or maps, networks, as well as in the representation of magnitudes and units of measurement and their relationship with the variables. On the other hand, the unification of these variables through formulas and the use of numbers instead of data should also be promoted	Concerning strategies, these refer to the set of organizational resources that lead to the resolution of problems and can generally be analyzed in two directions: forward when these strategies are conceived from what is given and lead to the generation of hypotheses, and backward from the existing information by analyzing possible intermediate results, using deduction that allows arriving at the given data.

Source: own construction based on Müller (1987, 2015).

When referring to mathematical models applied to the economy, the function of the macroeconomist, according to Tinbergen and Bos (1962), is to take care of the system of figures so that it does not present

any type of incompatibility. The functioning of the economic system implies the description of a mathematical model that is more complex than the ones we have. To ignore this is tantamount to denying the scientific study of the functioning of society.

It is also important to recognize the existence of stochastic variables that represent certain degrees of freedom that go against the assumption of determinacy. For the Nobel Prize winner, the management of a model consists of the orderly and complete administration of knowledge. However, Tinbergen and Bos (1962), resolve the question of what a model by is defining it as a certain number of elements, which are considered in their formal aspect, in this case, relegated to their purely economic content. The composition of a model implies:

Table VII. Elements of models.

Variables	Relations	Coefficients
A model includes several variables, which are subdivided into known or unknown or exogenous or endogenous variables that are susceptible to review in an analytical sense.	A model involves a series of equation relationships that allow the specification of existing links between variables. These are divided into definitions, equilibrium equations, technical and institutional equations, and behavioral equations. Each equation implies a set of reactions that may have a causal component.	These describe the intensity with which a variable is affected through the relationships described in the previous column. Generally, the additive constants are worked as coefficients. The coefficients give an account of the structure of the mechanism to be studied.

Source: Own construction based on Tinbergen and Bos (1962).

Heuristics continues to be one of the most important and debated concepts, although it has several different dimensions. However, for Romanycia and Pelletier (1985), this concept is articulated with the concepts of the uncertainty of the result, improvement of the performance of some process, and orientation towards decision making. On the other hand, individual heuristics, which relate to the ability to modify a particular task or subtask, have had a wide field of theoretical and conceptual development. Heuristics is, then, a method that allows the introduction of what can be called newly emerged variables.

In the analysis of the so-called collective interpretative frameworks, the role of social heuristics is very important, which mainly refers to deciding to consider a model that allows to operate efficiently, reducing cognitive costs (efficiency in thinking) and that is, in turn, a basis for social mobility. The works of Tversky and Kahneman (1974, 1979 & 1981), regarding cognitive judgment in heuristics, allows us to delve deeper into the consideration that economic actors have about information for decision making. Social heuristics can be considered as a set of archetypes or constructs that are collectively maintained by agents knowledgeable about the logic of the economic agent. They are, to a certain extent, micro-level institutions that maintain order in the market.

Social heuristics assume that decision making is guided by the search, evaluation, and selection of alternatives, which are finally reflected in a consensus heuristic, whose development is significantly influenced by the logic shared by social groups that must be aligned under a paradigmatic position that benefits the group at a given historical moment (Beamish and Woolsey 2010).

On the other hand, we can include the reflexivity heuristic that analyzes choices in terms of a simplification in the evaluation and selection of alternatives, rejecting alternatives that align with a paradigmatic position. Reflexivity would be in a purely individual terrain.

The heuristic model from Grubsky and Cumberwoorth (2014), recognized two different types. The first model is a model latent, which based in the origin of class and the destination of class. The perspective has a relation with the Marxism in the concept of class conscientiousness; however, the criticism about this concept is argued by Rothbard (1995), Von Mises (1957) Kolakowski (1974) and Van den Haag (1987), who established:

"But the distinctive point of Marxian theory is that class membership is decisive in determining

most and particularly political actions. This is patently wrong" (Van den Haag 1087, 28).

The argument with respect to the teleological process determined by the equality for the class of workers, supposes an increase in the levels of three categories: occupation, income, and wealth. The latency, keep to get an opportunity in the social ascent, and analyze this as a phenomenon tied to the desirable common social aspect, but itself represents a particular event, which represents a relationship with the different variables that allow a significant advance in the process of social ascent and that in a certain way is based on an occupation, an income, and a wealth of origin that in their combination will determine a social ascent that will entail a new occupation following training and technical appropriation processes acquired in development, with a higher income and a greater generation of wealth. The scheme presented by Grubsky and Cumberwoorth (2014) establishes two-way relationships between occupation and wealth.

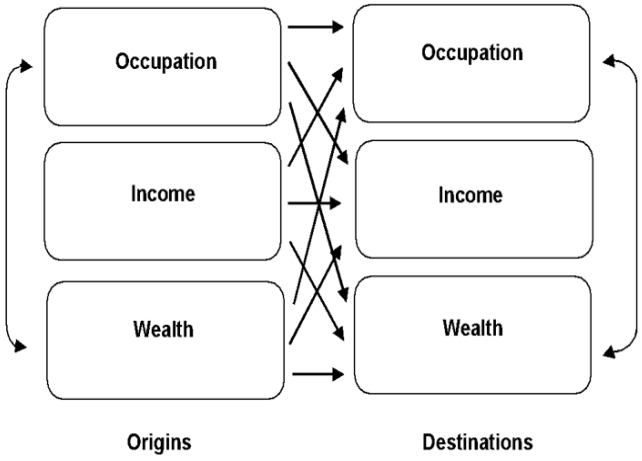


Figure 4. Heuristic latent class model.

Source: Grubsky and Cumberwoorth (2014).

Taking into account some of these previous aspects, the following is a presentation of the model.

Methodology

The model from Dongmei, Moulin and Wu

Pioneering work in epidemiology has been consolidated in the so-called Kermack-McKendrick hypothesis, which focuses on the prediction and distribution of cases occurring about infectious disease and its spread over time. Due to the importance of the model in the field of theoretical epidemiology, three basic papers for its development were published in 1927, 1932, and 1933. A substantial element of the model was the age of infection. Initially, the model used a set of partial differential equations that divided the population into infected persons in terms of their age and used the categories of susceptible (S), infected (I), and recovered or removed (R). Over time, the conditions would change, considering the following set of equations:

$$\frac{dS}{dt} = -\lambda S \tag{2}$$

$$\frac{\partial i}{\partial t} + \frac{\partial i}{\partial a} = (a)\lambda S - \gamma(a)i \tag{3}$$

$$(t) = \int_0^\infty i(a,t)da \tag{4}$$

$$\frac{dR}{dt} = \int_0^\infty (a)i(a,t)da \tag{5}$$

Where δ(α) is a Dirac delta function and represents the infection pressure:

$$I\lambda = \int_0^\infty (a)i(a,t)da \tag{6}$$

This formulation is equivalent to defining the incidence of infection as i (t, 0) = λS. In the case where the elimination rate ϒ(α)i and the transmission rate β(a) are constants at all ages, one can speak of prevalence in the epidemic dynamics, which gives rise to the so- called SIR model, which only considers infection and elimination events,

which are sufficient to describe a simple pandemic situation.

Based on the approaches of the previous model, proposed at the beginning of the last century, and subsequently improved with the inclusion of aspects related to immigration and perfect immunity, a similar model is proposed, which allows addressing the movement through time of different social layers and which also involves three categories. These categories attempt to account for inter-layer social mobility.

The pioneer's models of social mobility from McGinnis (1968), Boudon (1974) and other interesting manuscripts as Deary et al. (2005), have been based on innovative approaches to mobility analysis. In principle, the model attempts to obtain the number of persons of different levels of income that change between high, medium, and low income. First, we know the size of population N as a constant variable. The quantities are distributed in the three levels of income. We suppose the existence of population N, with three sets: high-income (HI), medium income (MI) and low-income (*LI*). Therefore:

$$N=HI+MI+LI \qquad (7)$$

Where:

N= Population.

HI = High-income.

MI = Medium-income.

LI = Low-income.

A mathematical model supported in social mobility argues that the people will be in a better position through the time. So other important variable is the time of change (t) between levels of income. Will be not good the change between a medium-income to low- income but is very desirable the change from low-income to medium-income. The ethic´s dilemma about the reduction of the high-income to medium income, is a topic tread in the re-distribution problem. One of the interests is known the change between people with low-income to medium income. The change through the time of the population with medium income, is represented by:

$$\frac{\partial MI}{\partial t} \qquad (8)$$

If the relation $\frac{\partial MI}{\partial t}$ growth the sign is positive and if the decrease the sign is negative. The relation between people with medium-income y people with low-income, generated motivation for reduce the social mobility (in fact the people with low-income search obtain a medium income). The relationship between LI and MI is the product between them (MI * LI). So, we say that the relation between MI and LI is proportional. This derivation will be expressed approximately to the relation between the populations with a medium income multiplied by the low-income. Then:

$$\frac{\partial MI}{\partial t} \sim (MI)(LI) \qquad (9)$$

However, not always the people of low-income, can get highest levels of income. There is a probability to get better levels of income. The model argues the existence of probability of social mobility, represented by φ, with a value between 0 and 1. So:

$$\frac{\partial MI}{\partial t} \sim \varphi(MI)(LI) \qquad \varphi = [0,1] \qquad (10)$$

If φ is equal to 1, all the people of low-income pass to better levels of income. The change through the time of the population of medium-income is represented by the inverse of the population (is important remember that N is constant). When the population of medium- income is big, this level of income is affected by the size of the population:

$$\frac{\partial MI}{\partial t} \sim \frac{1}{N} \qquad (11)$$

We have:

$$\frac{\partial MI}{\partial t} \sim \varphi \frac{(MI)(LI)}{N} \qquad (12)$$

The relation with respect to the population with *low-income* is expressed as:

$$\frac{\partial LI}{\partial t} \approx \frac{\partial MI}{\partial t} \qquad (13)$$

If we substitute the relation between *medium-income* and time, we have:

$$\frac{\partial LI}{\partial t} \varphi \frac{(MI)(LI)}{N} \qquad 14)$$

Now, we have interest in the population with low-income. An objective of the countries is less the size of population with low-income. Paradoxically, the size of the population in the low-income level is proportional to the amount of the population with medium-income. There are new people in the level with low-income, problem tied to the birth tax. So the change is proportional:

$$\frac{\partial LI}{\partial t} \approx \frac{\partial MI}{\partial t} = \varphi \frac{(MI)(LI)}{N} \qquad (15)$$

But, the social mobility is approximately determined by a mobility average (ω), which is multiplied for the population of low-income. Thus:

for the population of *low-income.* Thus:

$$\frac{\partial LI}{\partial t} \approx -\omega LI \qquad (16)$$

But we have a social mobility in relation with *medium-income,* so:

$$\frac{\partial LI}{\partial t} = \varphi \frac{(MI)(LI)}{N} - \omega LI \qquad (17)$$

This equation represented the positive change in the population with low-income, but it said nothing about the negative change. Here, negative is good and positive is bad. The number of populations from low-income to the level of medium income is expected to increase progressively.

It's important that the change in ω, be high in the population. However, we have some problems in relation with health, the sanitarian conditions, and nutrition among others. So, the result of ω, could be low. But the number of populations with low-income growths too with the population that transfer to levels of medium income to low income. The determination of φ and ω, is very important in

the definition of the social public policy as a mechanism of attack to the poverty. Finally, the number of populations with high-income, has the same proportion of the decrease of the population with medium and low income, so:

$$\frac{\partial HI}{\partial t} \approx -\Omega LI \qquad (18)$$

The sign is positive because the value always increases and in the worst situation is equal to 0. The following graphs show the expectations regarding progress over time for each of the social strata in the case of middle-income, low-income, and high-income. In the first case, the middle-income scenario assumes an increase that grows logarithmically over the long term or at least remains constant. The low-income scenario also assumes a logarithmic or linear growth, but it cannot be constant, if conditions are not guaranteed for large social strata to overcome poverty, and finally, the high-income layer is expected to remain constant or to have a decrease with a linear behavior with a negative slope, as its wealth must be distributed to the other strata. Exponential growth in the low-income stratum implies an enlistment and calibration of state policies to reduce poverty and improve their income; although it is an aspect that can be observed, it is not a "desirable" scenario, since it does not imply a significant exit of the low-income population from their condition:

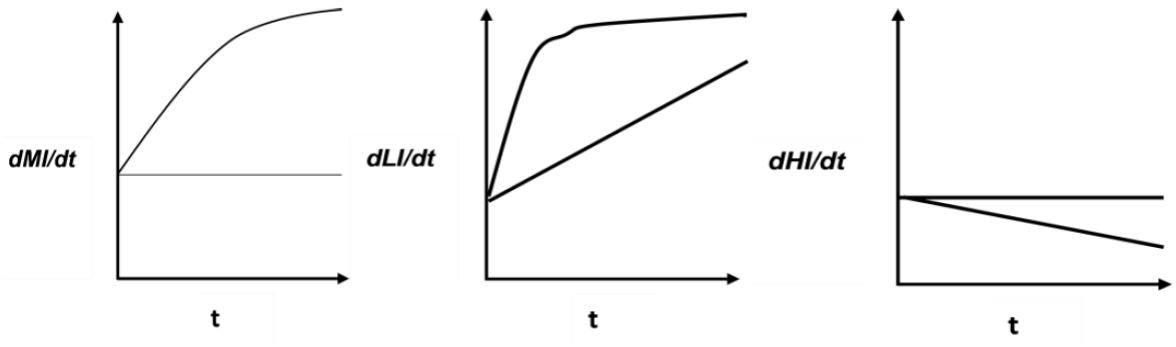


Figure 5. Behavior of the levels of income through the time.

Source: Own source.

These behaviors obey ideal states of social equity and imply collective acceptance of the effects of distribution. Redistribution is pyramidal and implies a detriment to the wealth of the upper strata of society, as opposed to a substantial improvement of the lower-income strata.

The following graph shows the relationship between social mobility and income distribution for some countries. Scandinavian countries show high levels of social mobility and better incomes, while countries such as the United States and the United Kingdom present a situation of low social mobility. In an intermediate state, we can find Canada, Germany, and France.

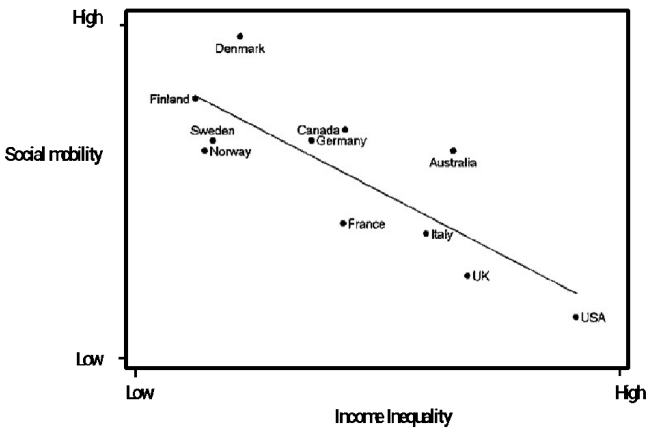


Figure 6. Social mobility is lower in more unequal countries.

Source: Wilkinson and Pickett (2009).

This box shows a very interesting inequitable relationship insofar as it shows not so high levels of equitable distribution in Anglo-Saxon countries, which is a determining factor insofar as they have historically been considered countries with high levels of production, being, in addition, the headquarters of the most important institutions of capitalism and with a widespread acceptance of ideas of economic liberalism and forms of social progress rooted in entrepreneurship, free enterprise and the application of scientific and technical advances to industrial and innovation processes.

The question that any analyst should ask in this regard focuses on why these societies turn out to be so unequal, despite the generation of wealth. Although the model offers some ideas regarding the idea of a status quo that permeates the national consciousness and produces a tacit acceptance of this inequality, it is important to recognize that social-democratic political models seem to show a greater interest in advancing policies of redistribution and promoting social mobility.

The failures in these systems are due to low levels of mobility average (ω), as an element of analysis and cause of the state's inability to suggest strategies for change in this situation, it should be suggested the orientation of population studies that account for how families move from one state to another in different generations.

Discussion

The conditions that this model shows to be analyzed in a broader perspective are due to how social strata can be significantly positively affected by novel heuristic studies in the field of Economics and that in interdisciplinary relation with other fields, allow overcoming the poverty trap, which many countries have experienced and whose traditional measures such as the Gini coefficient, have illustrated as trends that must be transformed based on profound proposals by the states concerning income redistribution. In words of Kamrava (1993):

"Social changes challenge the legitimacy of the political establishment by bringing into question those values on which the body politics is based. By nature, social change is inherently destabilizing process, upsetting the dominant values, relationships, and habits of the societies it affects. Such destabilization of values occurs not only in the social and cultural domains but in politics as well. In fact, the delegitimating of (all too often shallow) political values may be more acute than that affecting deeply rooted social and cultural norms. Social change brings into question the validity of those values with which the political establishment justifies its continued existence and its political agendas" (Kamrava 1993, 132).

For their part, heuristic models can make a fundamental contribution to the development of different explanations that can, in some way, contribute to significant progress in understanding how systematic errors have been made in decision-making in redistribution policy situations and to the analysis of different proposals to counteract this deepening of inequality, through proposals that emerge from other areas of development such as the model of epidemiological origin proposed by Dongmei et al (2014), and which is based on the pioneering models at the epidemiological level

proposed by Kermack and McKendrick (1927, 1933).

These findings provide models that can explain, from the point of view of social strata, a transition from low-income population sectors to higher income levels, thus generating greater progress in social mobility and an increase in well-being. The central point, in the explanation axis marked so far, is evidenced in the need for a substantial improvement and an increase in the value of social mobility coefficients (ω), which constitute a fundamental tool for measuring the efficiency and effectiveness of the effects of public redistribution policy.

This document focused on generating a proposal for a model to improve the measurement of redistribution, but it is expected that in the future, it could become a step forward for its application, validation, and reliability in different contexts that suffer from the problem of the poverty trap and that, as analyzed, in principle, present relatively high Gini coefficients in their historical measurements.

Appendix

Gini himself (1884-1965), in his 1946 book, -a Spanish version of 1952-, presents an appendix on an explanatory model on the relationship between species that he called explosion or flare of the devouring species.

If this model could be taken to the analysis of two social strata, one of low income LI and the other of high income HI, it could be shown that the relationship between the two social strata is balanced from the stationary point of view either from a periodic or an asymptotic point of view.

In this sense, the social conditions can be given so that the high-income layer takes advantage of and benefits from the exploitation of the low-income layer and thus presents a constant growth rate or growth coefficient. From this perspective, quantitative relationships can be analyzed through the following system of equations:

$$\begin{cases} \frac{dN_1}{dt} = \gamma_1 N_1 - \delta_1 N_2 \end{cases} \qquad (1)$$

$$\begin{cases} \frac{dN_1}{dt} = \gamma_2 N_2 \end{cases} \qquad (2)$$

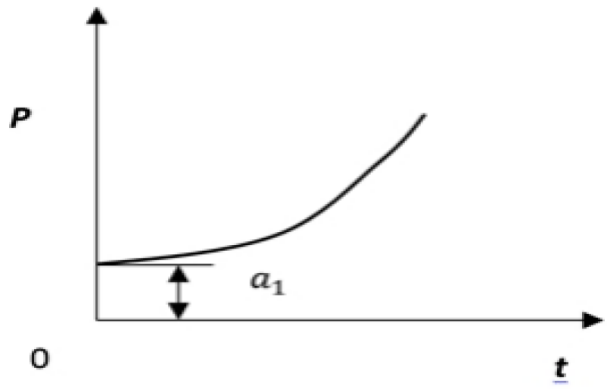
Where N1 corresponds to the high-income individual, while N2, represents to low-income individuals, γ_1 and γ_2, correspond to growth coefficients and δ_1, is a decreased coefficient, all three coefficients being positive.

The System can be solved as

$$N_2 = a_1 e^{\gamma_2 t} \qquad (3)$$

$$N_1 = a_1 e^{\gamma_1 t} + \frac{\delta_1}{\gamma_1 - \gamma_2} a_2 (e^{\gamma_2 t} - e^{\gamma_1 t} \qquad (4)$$

In this case a_1, and a_2, are the number of Individuals of every social strata in the initial moment. The equation 3, has an exponential behavior, which can be seen in the following figure:



Equation 4, on the other hand, can have three types of development, depending on the growth and decay coefficients (γ_1 and γ_2, as well as the initial values (a_1, and $a_{2)}$:

Case A, under the conditions of

$$\begin{cases} \gamma_1 - \gamma_2 > 0 \\ \frac{a_1}{a_2} \geq \frac{\delta}{\gamma_1 - \gamma_2} \end{cases}$$

Case B, under the conditions

$$\begin{cases} \gamma_1 - \gamma_2 > 0 \\ \frac{\delta_1}{\gamma_2} < \frac{a_1}{a_2} < \frac{\delta_1}{\gamma_1 - \gamma_2} \end{cases}$$

Or under the conditions of

$$\begin{cases} \gamma_1 - \gamma_2 \leq 0 \\ \frac{a_1}{a_2} > \frac{\delta_1}{\gamma_1} \end{cases}$$

Case C, under the conditions

$$\begin{cases} \frac{a_1}{a_2} < \frac{\delta_1}{\gamma_1} \end{cases}$$

In the first case, the social strata N1 of low income LI, growths ad infinitum¸ as an exploitative layer serving the high-income social layer of N2, in which case income will tend towards infinity. However, it is clear that both social layers cannot cross a certain limit specifically given by the social context.

For case B, the number of individuals in the low-income stratum grows to a maximum number and then decreases, with the hope of their disappearance (ideal situation in an economy with absolute redistribution).

For case C, the number of individuals in the low-income social stratum decreases continuously until it reaches zero in a defined time. In cases B and C, the high-income social stratum would have an increase in the number of individuals, and, at the same time, exploitation of low-income individuals would be assumed, which is the ideal situation.

Thus, the proposal to establish a stationary equilibrium between high and low-income social strata is not, from an ethical point of view, desirable and public redistribution policies cannot allow exploitation of the high-income strata over the low-income ones. In this sense, the system:

$$\frac{dN_1}{dt} = \gamma_1 N_1 - \delta_1 N_1 N_2 \quad (5)$$

$$\frac{dN_2}{dt} = -\gamma_1 N_1 + \delta_2 N_1 N_2 \quad (6)$$

The growth of $\frac{dN_1}{dt}$, of the high-income stratum or social stratum is given by $\delta_1 N_1 N_2$, which is tending to decrease, while the growth of the $\frac{dN_2}{dt}$, of the low-income social stratum is increasing in a society without redistribution $\delta_2 N_1 N_2$.

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