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Deconstruction of the logarithmic function: Teacher training

Deconstrucción de la función logarítmica: formación de profesores

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Abstract: The teaching and learning of the logarithmic function have been a purpose of study in mathematics education, both teachers and students find the understanding of this concept challenging. This has been detected or analyzed through interviews and work proposals in the classroom, among others. In this case, we consider the need for interventions that directly involve teachers. The ideas from the didactics of mathematics regarding the Basic Models by which underlying hypothesis are taken up, that considers the unique approach to teaching the logarithmic function as the inverse of the exponential function is not enough and it can contribute to challenges in learning. In this project when approaching teacher training, a Hypothetical Learning Path is elaborated, so that teachers can deconstruct this concept in their initial or continuing training process. Taking advantage of the results of inquiries in mathematics education, documentary research with phenomenological perspective is performed and, qualitative data are processed with an interpretative and descriptive analysis using the concept of Hypothetical Learning Path. For the deconstruction of the concept of the Hypothetical Path, there are objectives, tasks, questions, and experiences proposed; and the practicing or training teachers are required to characterize these functions through various mathematical elements. Based on the study and analysis of the publications of the academic community, the proposals and findings are selected and used in part or in full allows the teacher to delve through the different Basic Models and at the same time researchers indirectly embedded in the role of curators in education.

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Palabras clave

Precálculo, Formación De Profesores, Función Logarítmica, Rol De Curador, Trayectoria Hipotética De Aprendizaje.

Resumen: La enseñanza y el aprendizaje de la función logarítmica ha sido objeto de estudio en Educación Matemática, evidenciándose diversas dificultades en la comprensión de este concepto tanto por parte de los docentes como de los estudiantes, las cuales han sido detectadas o analizadas a través de entrevistas y propuestas de trabajo en el aula, entre otros. En este caso se procede desde la necesidad de intervenciones que impliquen directamente a los profesores, se retoman las ideas desde la didacta de la matemática respecto a los Modelos Básicos y con la hipótesis que subyace, concerniente a que el enfoque único en la enseñanza de la función logarítmica como la inversa de la función exponencial, no es suficiente y además éste puede coadyuvar a generar dificultades en su aprendizaje. Así, en este proyecto al abordar la formación de profesores, se elabora una Trayectoria Hipotética de Aprendizaje, de manera que ellos tengan la oportunidad de deconstruir este concepto en su proceso de formación. Aprovechando los resultados de indagaciones en educación matemática, a partir de una investigación documental y con perspectiva fenomenológica, se procesan datos cualitativos con un análisis descriptivo interpretativo recurriendo a la noción de Trayectoria Hipotética de Aprendizaje. En esta Trayectoria Hipotética de deconstrucción del concepto se proponen objetivos, tareas, preguntas y experiencias; con una exigencia cognitiva que les permita a los profesores en ejercicio o en formación inicial, caracterizar estas funciones mediante diversos elementos matemáticos. Así, a partir del estudio y análisis de las publicaciones de la comunidad académica, se seleccionan y se hace uso en parte de las propuestas y hallazgos, que de acuerdo con los análisis de los investigadores abonan una posibilidad de que el profesor pueda transitar por los diferentes Modelos Básicos y al mismo tiempo los investigadores estén insertos tangencialmente en el rol de curadores en educación.

Introduction

This theoretical proposal gives continuity to the investigations of Vargas and González (2007) and Vargas, González y Vargas (2020), where it has been shown that in teaching the logarithmic function, teachers model the inversion mechanism in their classes, similar to the finding of authors who consider the resource of teaching the logarithmic function as the inverse of the exponential function challenging.

On the other hand, open question around this concept suggests other paths in this line of research. The publication of Weber (2017) takes up the challenges that students may have to understand these concepts and after a synthesis of the historical-epistemological development, like the one exposed by González and Vargas (2007), it presents its approach in a teaching process that would allow a structural and procedural understanding of said concept.

We start with the 'Basic Models' (Vom Hofe et al. 2006, cited by Weber, 2016) using a theoretical construction that describes how mathematical concepts can be made understandable for students based on different interpretations; the study of the Basic Models of Logarithms (Weber, 2016, 2017) and the review of the state of the art of this topic. A proposal is made for the training of mathematics teachers that answers the question: What can be an alternative Hypothetical Learning Path of the logarithmic function that allows future teachers to interact with the results of research in mathematics education and deconstruct the concept of logarithmic function?

Background

On the other hand, in the study by Weber (2002) there is a research regarding the challenges that students face when carrying out the mental constructions necessary for understanding the concept of logarithmic function, recognize it as an inverse function of the exponential function, address the conflicts generated by the peculiar notation of this function, as well as establishing the conversions between the graphic and algebraic representations associated with the concept and the applications of the concept in different areas, which is associated with the problem solving process.

Several international researches have analyzed the challenges of students (Kenney and Kastberg, 2013), or have studied the use of new teaching tools such as graphing calculators or software, that use dynamic geometry and computational algebraic systems (CAS) (Koştur and Ayşenur, 2017), or the epistemology of the concept, emphasizing the relationship between arithmetic and geometric progressions by Vargas, Pérez and González (2011), or the fact that the characterization of this function is found in the constant nature of the subtangent (Dennis and Confrey, 1997). In addition, there are also inquiries about the knowledge of teachers or teaching practices (Getenet and Beswick, 2014; Vargas and González, in press, 2017) and the design of didactic tools for teacher training (Bocanegra, Galeano and Huérfano, 2013).

In a similar way, there are theoretical proposals, such as that of Weber (2017), where besides recognizing the challenge of the logarithmic function in the teaching process, it also points out the relevance in the modeling of phenomena.

According to the author, all the above gives way to new forms of theoretical and practical inquiries about this concept and the learning processes, and in this way the present investigation was conducted. Thus, an approach for the understanding of the logarithmic function for teachers in training a - Hypothetical Learning Path - retaking and transforming one of the arguments of Weber (2017), following a concept where "logarithmic function" is encapsulated when the discourse about logarithms, as numbers and operators, blends with the discourse of logarithmic graphs and is understood as an object.

In this case, the underlying hypothesis is that the unique approach of the logarithmic function as the inverse of the exponential function is not enough, and it can also contribute to generating difficulties in learning (Weber, 2017; Vargas, González and Pérez, 2011). This situation can be addressed in teacher training, so that teachers have the opportunity to deconstruct this concept based on the Basic Models and collaborative learning, which can be key mechanisms that enhance the exam of mathematical elements that allow understanding the logarithmic function as an object that is vital in mathematical modeling.

Theoretical references and state of the art

The theoretical references that guided the research are presented and described below, there are the basis for the selection and design of the activities proposed for teachers in training.

Deconstruction of mathematical knowledge: For this research, deconstruction of mathematical knowledge will be the decomposition of the mathematical elements of a concept for its understanding, and the integration that occurs when teachers face the task of designing learning activities, that is, the requirement to include different sources of their professional knowledge (mathematical, didactic, contextual of their classroom, curricular, among others) (Cabrera and Cantoral 2013; Krieger, 2004).

“Deconstruction not in the sense of disolve or destruct but in the sense of analyze the sedimented structures that form the discursive element, the philosophical dicursivity that we think. This analyzes across by the language, by the occidental culture and by the set that defines our belonging to this history of philosophy”. Jacques Derrida (1930-2004).

Hypothetical Learning Path (HLP): refers to the objectives for student learning, the mathematical tasks that will be used to promote their learning and the hypotheses about the learning process of a certain concept (Simon, 1995).

While the teacher's goal for student learning provides direction for the other components, the selection of learning tasks and assumptions about the student's learning process are interdependent. Tasks are selected based on assumptions about the learning process and assumptions about the learning process are based on the proposed tasks.

This construct is based on the following assumptions:

1. The construction of a Hypothetical Learning Path is based on the understanding of the current knowledge of the students who will receive the instruction.

2. A Hypothetical Learning Path is the vehicle for planning the learning of specific mathematical concepts.

3. Mathematical tasks provide the tools to promote the learning of specific mathematical concepts and are therefore a key element of the instructional process.

4. Given the hypothetical and inherently uncertain nature of this process, the teacher will be forced to systematically modify every aspect of the Hypothetical Learning Path.

Therefore, an HLP is the path that a student follows to achieve learning and is built through a design investigation to identify the most likely steps that he takes as he develops his initial mathematical ideas until he reaches the formal concepts (Maloney and Confrey, 2010).

In this notion, based on constructivist learning, the HLP is not unique and requires continuous readjustment depending on the results obtained from its implementation.

Basic Models: Are a theoretical construction to capture what it means to make the concepts accessible or understand them, they are called "Grundvorstellungen" (Vom Hofe and Blum, 2016). In short, a Basic Model of a concept is an interpretation of it in a context in which students are likely to have more experience, in addition, each Basic Model has a certain explanatory power, which allows students to argue and solve certain tasks.

In the case of logarithms, taking as a reference several of the investigations that have been carried out in the field of Mathematics Education, Weber (2017) proposes to make use of familiar contexts for meaningful interpretations of concepts, integrating to them the notions of operationality that corresponds to the processes and structure that refers to the objects (Sfard, 1991).

That is to say, this author considers that when applying the two theories; the role played by accessibility and the two ways of conceiving the concept will be taken into account: operationally or structurally. Therefore, depending on how it is approached or acted upon, the mathematical content can be thought from an operational or structural perspective, and by moving on these two, more meaning can be given to the concepts.

Based on the above, the author proposes to examine his Basic Models, identifying which one allows a discourse about processes and actions and which one about states and objects. In that research they were called Operational Basic Models and Structural Basic Models.

We take verbatim the following Basic Models (BM) proposed in Weber (2016; 2017):

● (BM1) Logarithms as a multiplicative measure: The logarithm of a number b indicates how often the base a fit the number b as a factor.

Or else, (BM1') the logarithm of a number b (to the base a) indicates how often the number b has to be divided by the base a to produce 1. This model is also called the Basic 'Repeated Division' Model.

● (BM2) Logarithms as counting the number of digits: The (common) logarithm of a number b finds the number of digits of b needed to represent b in positional notation, minus one.

● (BM3) Logarithms as hierarchy level decrease: the logarithm of an expression reduces third-level operations (powers, roots) to second-level operations (multiplications, divisions) and reduces second-level operations to first-level operations (additions, subtraction).Thus, the understanding of the logarithm in this model (BM3) "decrease the hierarchy level" is considered an operational conception (Basic Model that is not oriented towards numbers but towards algebraic expressions), examining the logarithm as an operator and not as a number.

● (BM4) Logarithms as inverse exponents: The logarithm of a number (or expression) erezovski(or expression). This is considered a structural conception, it resorts to the notion of inverse (this model refers to another concept, the exponent concept either numbers or expressions).

On the one hand, according to Weber (2017) the models (BM1), (BM1') and (MB2) refer to logarithms as numbers. Regarding the model (BM3) refers to logarithms as operators, while (MB4) refers to numbers and operators.

Additionally, according to the author, it is assumed that the concept of "logarithmic function" is reified as soon as the discourse about logarithms as numbers and operators merges with the discourse of logarithmic graphs, as soon as they "become mere representations" of the newborn object (Sfard, 2018, p.122).

In this sense, Models (BM1) and (BM1') are more closely related to the operational conception of logarithms, since they generate results, while (MB4) is more closely related to their structural conception and their structure refers to another concept, that of exponent. Weber (2016) highlights that, from an epistemological point of view, conceiving logarithmic functions as inverse functions of exponentials reflects the structural vision of experts who have reified their experiences. As for the two remaining Basic Models (BM2) and (BM3), they can be potentially useful to unite the operational and structural conceptions of logarithms, emphasizing the effects of logarithms.

Methodology

Taking the training of mathematics teachers as a research object, this object gives way to concerns about learning in the professional development processes of educators; in the case, referring to the understanding of the logarithmic function object.

The design of this research, flexible and located, is based on the notion of hypothetical trajectory, being this organized from the assumptions provided from the operational basic models and structural basic models. These models provide the hypothesis about the learning process and are interdependent with the tasks that are selected. It is kept in mind that the resource for obtaining the information, for the tasks, will be a previous review of the state of the art of publications in Spanish with proposals for the teaching of logarithms and logarithmic functions.

To address the hypothesis underlying the question under study, a qualitative descriptive investigation will be carried out, with which it is expected to obtain a teaching and learning strategy - Hypothetical Learning Path - that impacts on the training and practice of the teachers. In a second stage, it will be implemented and validated in programs of different universities that are responsible for such training.

It should be noted that both this inquiry and the resulting proposal are not unrelated to the development of information and communication technologies, specifically the Internet, which has stimulated a growth of digital information in such that it requires for the information to be classified. This progress alludes to the concept of digital curators.

For the concept of curator, it is an art, is conceived as a specialist who is responsible for: selecting, caring for, exhibiting, evaluating, preserving the collections of certain cultural institutions (Garzón, 2016). In this sense, content curation is associated with the practice of art´s curator. At the beginning of the 21st century, the term curatorship begins to be adopted for digital information, hence content curation is understood as "the interactive act of investigating, finding, filtering, organizing, grouping, integrating, editing and share the best and most relevant content on a specific topic in a meaningful online digital collection, which could be important for a group of people whose sense of learning can be updated around that topic" (Posada, 2013, p.3).

In the educational field, according to several authors, the teacher is one of those summoned to carry out and use digital content curation (Juárez, Torres and Herrera, 2017). Thus, its curation process requires an analysis or discussion that transcends the educator's disciplinary knowledge and requires pedagogical knowledge in curating content, so that it is relevant and favors the incorporation of content into the classroom with disciplinary and pedagogical correct criteria (Garzón, 2016).

From this process, as of the findings that are contemplated in the antecedents and the Basic Models enunciated in the theoretical framework, some of the steps in this investigation were:

- The literature review started with the research of Vargas and González (in press); related to the teaching and learning of these concepts, and attention was focused on research results referring to the knowledge of teachers on this topic, to which was added the review of studies concerning historical development from the point of view of its use in the teacher training, in Vargas (2019; 2020).
- The documents that presented a proposal to study logarithms or the logarithmic function were selected, which allowed to illustrate and discuss tasks in coherence with one of the Basic Models.

- A diagnostic instrument was designed and validated aimed at detecting the various approaches, manifested by teachers in training, concerning the Basic Models.

- 25 isolated ‘exercises’ were proposed that were created, with the documentation, to correspond to one of the Basic Models of logarithms.

- Mathematical tasks were designed, based on the Basic Models referenced in the theoretical framework.

- Sequences of Basic Models were identified that could be regrouped in a session and thus, the subsequent step was to create three categories and eliminate tasks that could be repetitive, choosing to leave a reduced number of documents that would allow diversity of approaches and in turn make possible the in-depth study of the aspect that each of them wishes to study.

In last phase of theoretical validation processes of the designed activities, proceeded to the triangulation of the information and integration of the aspects into tasks for a proposal of a Hypothetical Learning Path that, from the examination of two researchers, allows the deconstruction of the concept.

For example, in the classification that refers to the transition from the operational to the structural conception, the thesis of, Bocanegra, Galeano and Huérfano (2013) was studied, extracting, after a triangulation process, only for the MB3 model, it is proposed to develop a task, from pages 81 to 88, in which, through the use of tables, from the relationship between an arithmetic and geometric sequence, properties are identified and some regularities and patterns are determined.

Another of the selected publications is the study of the vignettes by researchers Vargas, González, and Vargas (2020), concerning the classroom management of a pre-calculus teacher and analyze from there, the tasks in terms of resources representation and relationships between concepts, cognitive demand of the questions and exercises that are developed in the classes, when examining the properties such as the linearization of the logarithms.

These isolated tasks are linked together, in such a way that in the proposal a sequence is established between them, starting with the task called activity and then expansion activities.

Results

The following is the proposed Hypothetical Learning Path for the deconstruction of the logarithm and logarithmic function aimed at mathematics teachers in training. This has a structure where the objectives are presented, the mathematical element inserted in the Basic Models involved, an introduction, the tasks, and some in-depth activities.

In this Hypothetical Path, it is intended that, in a cross-sectional way, all tasks, for the students in training for teachers, go through the historical-epistemological development of the concept and through the mathematical practices of identification and use of regularities and patterns.

The general structure of the tasks in this Path is in the table I.

Table I. Structure Of The Tasks

TASK 1	-Basic repeated division model.	MB1	Operational conception	Logarithms as numbers.
Expansion activities	-Basic model like counting the number of digits. -Common logarithm.	MB2	Operational conception	MB2 to blend the operational and structural conception.
TASK 2	-Decrease in hierarchy. -Processes of identification of regularities.	MB3	Operational conception	Logarithms as numbers and as operators.
Expansion activities	-Properties of logarithms through the relationship between the terms of the arithmetic and geometric sequences. -The logarithmic scale.	MB3	Operational conception Algebraic expressions Structural conception	MB3 to blend the operational and structural conception. MB4 to appropriate structural conception.
TASK 3	-Construction of the graphical representation of the logarithmic function. -Variation and growth in functions with various bases.	MB4	Structural conception	Covariation
Expansion activities	Logarithmic and exponential function as inverse functions.	MB4	Structural conception Analytical expression	Properties

Hypothetical Learning Path of the logarithmic function in the training of mathematics teachers

Objectives for learning

General goals

Deconstruct the concept of logarithm and of logarithmic function, considering Weber's Basic Models of logarithms (Weber, 2016), taking up historical-epistemological aspects and an approach to LKT- Learning and Knowledge Technologies - to give greater meaning to these mathematical objects

. **Specific goals**

Appropriate an algorithm that allows defining and understanding logarithms in a way that is close to the arithmetic context and different from the usual one.

Identify characteristics and properties of the logarithmic function, from the knowledge of the epistemological historical development and through the analysis of documents from Mathematics Education, using graphic representations and geometric models that allow the construction of these (Interpolation and geometric models).

Incorporate the mathematical practices of identification and use of regularities and patterns to the study of the characterization of functions, both in their properties and in their characteristic covariation.

Study and solve a diversity of situations that allow an approach to models of logarithms both operationally and structurally.

Math tasks to promote learning

Next, the sequence of tasks for learning the logarithmic function based on the Basic Models of logarithms is presented.

Definition of a new operation and construction of the concept of logarithm

Introduction

From Weber (2016; 2017), it is considered that the teaching of the concept of logarithm and the logarithmic function can be started from the Basic Model 1, given its proximity to the context of school mathematics and since its definition allows to determine logarithms through an algorithm.

In this sense, this activity seeks to build a definition of a logarithm, as an operation, from another operation such as division, a method that will allow us to understand and justify some of its properties and give more meaning to the logarithms.

Division is an operation that answers how many times a number of fits a in a number b, which can be obtained through the development of repeated subtractions, so that, to the number b it subtracted several times a until getting cero.

Example: 8÷2=?

$$8-2=6$$

$$6-2=4$$

$$4-2=2$$

$$2-2=0$$

Counting the number that was subtracted 2, it is concluded that, 8 divided 2 it is equal to 4.

If the division is not exact, we must speak of a remainder.

Example: 15÷2=?

$$15-2=13$$

$$13-2=11$$

$$11-2=9$$

$$9-2=7$$

$$7-2=5$$

$$5-2=3$$

$$3-2=1$$

15 divided 2 it is equal to7 and the residue is1. However, if you want to get the complete result of the division, you must multiply the remainder by 10 and repeat the subtraction process, which will result in a decimal number.

$$1 \times 10 = 10$$

$$10 - 2 = 8$$

$$8 - 2 = 6$$

$$6 - 2 = 4$$

$$4 - 2 = 2$$

$$2 - 2 = 0$$

Thus, the result of diving 15 between two is equal to 7,5.

Consider now the following operation:

$b \Delta a$, indicate how many factors a are required to obtain the number b, its result is achieved through the development of repeated divisions, so that the numberb it is divided serveal times a until obtaining 1.

Example: 8 Δ 2=?

$$8 \div 2 = 4$$

$$4 \div 2 = 2$$

$$2 \div 2 = 1$$

As the number of factors 2 that is required to obtain8 is 3, it is concluded that,

8 Δ 2 =3

Let us consider 25 Δ 2

$$25 \div 2 = 12{,}5$$

$$12{,}5 \div 2 = 6{,}25$$

$$6{,}25 \div 2 = 3{,}125$$

$$3{,}125 \div 2 = 1{,}5625$$

The number of factors is four (4), however as there is a surplus of 1,5625, we will give 2 more figures of accuracy. For this, the surplus value is raised to 10.

$$1{,}5625^{10} = 86{,}736$$

$$86{,}736 \div 2 = 43{,}368$$

$$43{,}368 \div 2 = 21{,}684$$

$$21{,}684 \div 2 = 10{,}842$$

$$10{,}842 \div 2 = 5{,}421$$

$$5{,}421 \div 2 = 2{,}7105$$

$$2{,}7105 \div 2 = 1{,}355$$

6 factors.

$$1{,}355^{10} = 20{,}9023$$

$$20{,}9023 \div 2 = 10{,}4511$$

$$10{,}4511 \div 2 = 5{,}2255$$

$$5{,}2255 \div 2 = 2{,}6127$$

$$2{,}6127 \div 2 = 1{,}3063$$

4 factors.

The process is repeated until obtaining 1, but for this operation it would be enough to give two decimal places, therefore, 25 Δ 2 = 4,64. It is clarified that, as in the division, when raising to the 10, the values obtained correspond to the decimal part.

Activity

Based on the definition of the operation b Δ a, answer each of the following questions.

Determine:

19683 Δ 3=
32 Δ 5=

Mathematically the operation $b \Delta a$, known as the logarithm to base a of b and it is represented $\log_a b$.

From this new note rewrite:

$$35 \,\Delta\, 6$$
$$120 \,\Delta\, 10$$

Using the repeated division process calculate:

$$\mathrm{Log}_5\ 15625 =$$
$$\mathrm{Log}_6\ 45 =$$

Why is it correct to state that?

$Log_a\, a^n = n$

Make three examples and then give a general justification.

Deepening activity

There are a variety of ways to approach the concept of logarithm.

Historically, the concept of logarithm arose from the relationship that can be established between a geometric progression of ratio r and an arithmetic of difference d (González y Vargas, 2007). From this relationship, and under the need to facilitate arithmetic calculations, the logarithm was. When conducting the study of this document, it establishes the analyzes presented between the sequences, how the common logarithm can be understood, with an approximation through the number of digits.

Logarithm of a product and logarithm of a quotient

Introduction

The study of mathematical concepts in the classroom, such as social and cultural creations, is an interesting aspect for teachers and students to cultivate.

Hence, the study of the historical and epistemological development of the logarithmic function intends, among others, to highlight to trainers, the various definitions of mathematical concepts and the changes they have undergone, such as logarithms; allowing them to be analytical objects. Thus, the Cauchy definition stands out in which a logarithmic function is the only continuous solution Φ of the functional equation Φ (x . y) $= \Phi(x) + \Phi(y)$ ($x> 0$, y> 0).

The idea is to contextualize a work strategy in the classroom through the mathematical practices of identification and use of patterns. It begins with a reading concerning the innovation of teaching in the pre-calculus courses, through a cultural approach mediated by the analysis of pre-Columbian art, a Mathematical Education study by Vargas, Vargas and Cáceres (2020). Specific examples can be seen in Vargas and Vargas (2019) .

Activity

Considering that in task 1 it is requested to consult the thesis Cano and García (2016), it is requested to develop workshops three and four of this thesis (annexes 3 and 4) for the construction of the properties.

Analysis and construction of graphs of logarithmic functions

Introduction

As known, the concept of logarithm arises from the relationship between a geometric progression and an arithmetic, with respect to the logarithmic function, its construction is possible by locating a geometric progression of ratio b in the axis x and an arithmetic of difference 1 in the axis y, however, the question arises about how to locate more points of this function to obtain a more exact graph, and in this way find the logarithms of other values in the domain.

Vargas, Pérez and González (2011)[3] , in their article “The logarithm: How to animate a point that relates to a geometric progression and an arithmetic?” suggest obtaining new points from the construction of geometric and arithmetic means, which allows finding new logarithms and also obtaining a graph with greater accuracy. From these new points the idea is to establish some characteristics of the logarithmic function and through interpolation to make an approximation to continuity, therefore, it is requested to consult the digital repository of documents in mathematics education.

Here is an example for the function $f(x)= log_2(x)$.

The points *A,B,C,D,E* and *F* have coordinates x in geometric progression of ratio 2 and the y in arithmetic progression of difference 1, these points correspond to the logarithmic base function 2, to obtain other points of this function, the geometric mean and the arithmetic mean of already known values must be constructed, the Figure 1 illustrates the geometric mean between 8 and 16 (point *G*) and the arithmetic mean of their respective logarithms (point *S*).

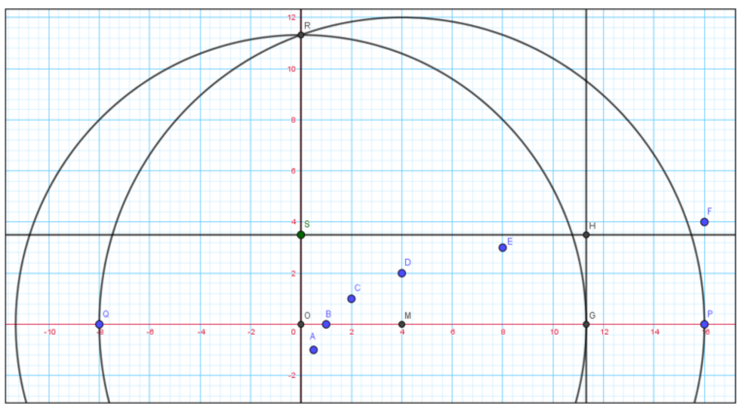

Figure 1: Construction of the logarithmic function using geometric and arithmetic means. Source: own.

Construction of the geometric mean between 8 and 16 on the Cartesian plane

Mark the origin with a point *O*.

Build a segment *OP* over the measurement x axis 16towards the right of the origin.

Build a segment *OQ* over the measurement x axis 8towards the left of the origin.

Determine the average point M between the points *Q* and *P*.

Construct a circumference with center M and radio *QM*.

Trace the perpendicular to the axis x by the point *O* and determine the intersection R between these circureference and the perpendicular, the segment *OR* corresponds to the geometric average 8 and 16.

By means of a circle copy the measure of this segment on the axis x by intersection point *G*.

To obtain the arithmetic mean of the respective logarithms, that is,3 and 4,the average point is constructed S amongst these values over the axis. y. Finally, draw two perpendiculars to the axes x and y by the points G and S call H to its intersection, this point is on the logarithmic function and indicates that the: Log_2 11,31 =2,5.

Activity

From a geometric progression of ratio 3 and an arithmetic difference 1 and using geometric and arithmetic means, construct a chunk of the function $f(x) = \text{Log}_3 x$, for the interval (0,27] over the axis x.

Using the properties of the product and quotient of logarithms and from the logarithms found with geometric and arithmetic means, devise a method to obtain new points inside and outside the given interval, which will allow to better extend and approximate this logarithmic function.

Join these points using segments and then compare them with the logarithmic function that the program generates. How close is the graph of the function?

Consult the definition of geometric and arithmetic mean, and from these propose an arithmetic method to find logarithms of values other than those of the geometric progression.

Build some relationships:

With the help of the program, construct the graphs of the functions $f(x) = \text{Log}_2 x$ and $g(x) = \text{Log}_{1/2} x$.

How is the variation in the axis x of the function of the base 1/2?

How is the variation in the axis y of the function of the base 1/2?

Justify why the base function 1/2 is decreasing.

Establish a general relation for base functions b and its multiplicative opposite 1/b.

An activity that is considered relevant given of analysis and synthesis requirements, in the teacher training process, consists of making a 20 minute collaborative presentation in Padlet, that allows identifying the contributions of each student, where the team exposes the characteristics of logarithms, that have been previously studied through examples, focused on the logarithmic scale, of the article presented by Vargas, Chavés and Jaimes (2018a) or through the transformation of said examples.

To conclude:

In the general scheme of the path the impact of research in mathematics education is enhanced, bringing it closer to professional training and in turn it tends to examine teaching through research by supporting the study directly from the documentation.

A proposal that highlights the need to involve the knowledge of the teacher, strategies concerning the mathematical practices of identification and use of patterns within the Basic Models is presented, this is done through the selection of an online article, that connects said practices with the study of the logarithmic function, among other reference documents.

Although this Hypothetical Path traced by the researchers matches aspects stated in the perspectives of mathematics education, it also accounts for the challenge of teacher educators in a job that can well be identified with that of content curator; in the analysis and selection of materials, adding to this the creation of tasks with identification of the cognitive demand and the design of a structure to articulate elements so that the student can access these contents or experiences.

The staging of this Hypothetical Learning Path is essential to validate it, which is an additional step to the contribution that is established to the outline of the theoretical proposal of the Basic Models stated.

As this Hypothetical Learning Path was elaborated to allow a deconstruction, in the initial teacher training, it was decided to start using the operational conception (BM1') and then we moved to a more structural one (BM4), selecting tasks that represent a cognitive requirement, that allow to study and 'mix' all the Basic Models.

A rigorous analysis of the advantages for students in teacher training is pending, which interacts with fully contextualized materials in the teaching and learning of the logarithmic function, at the same time that they are basing their knowledge of the mathematical object.

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