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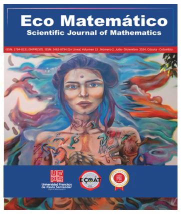

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Strategies for solving problems of multiplicative structure in elementary education

Estrategias de resolución de problemas de estructura multiplicativa en educación primaria

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Keywords

Elementary education, Multiplication, Natural numbers, problem solving.

Abstract: This article examines the approaches elementary school students use to address multiplicative structure problems, including isomorphism of measures, multiplicative comparison, and product of measurements. The research utilized a quantitative methodological strategy, employing a questionnaire containing eight problems involving natural numbers to assess both the success rates of the students and the strategies they employed. The findings reveal that students exhibited a lower level of success when dealing with problems involving product of measures, largely due to their difficulty in recognizing the uniform structure underlying these problems. While algorithms emerged as the predominant strategy among students, the study also identified the use of various alternative strategies, which varied according to the magnitude of the quantities involved in the problems. This research highlights the need for improved instructional methods to help students better understand and apply consistent problem-solving structures across different types of multiplicative problems

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Palabras clave:

Educación primaria, Multiplicación, Números naturales, Resolución de problemas.

Resumen: Este artículo examina las estrategias que utilizan algunos estudiantes de educación básica primaria para abordar problemas de estructura multiplicativa, incluyendo isomorfismo de medidas, comparación multiplicativa y producto de medidas. La investigación utilizó una estrategia metodológica cuantitativa, empleando un cuestionario que contenía ocho problemas que involucraban números naturales para evaluar tanto las tasas de éxito de los estudiantes como las estrategias que empleaban. Los resultados revelan que los estudiantes mostraron un menor nivel de éxito cuando se enfrentaron a problemas que implicaban producto de medidas, en gran parte debido a su dificultad para reconocer la estructura uniforme subyacente a estos problemas. Aunque los algoritmos resultaron ser la estrategia predominante entre los estudiantes, el estudio también identificó el uso de diversas estrategias alternativas, que variaban en función de la magnitud de las cantidades implicadas en los problemas. Esta investigación pone de manifiesto la necesidad de mejorar los métodos de instrucción para ayudar a los estudiantes a comprender mejor y aplicar estructuras coherentes de resolución de problemas en distintos tipos de problemas multiplicativos.

Introduction

Multiplicative problems developed in elementary school have a structure that is determined by the role that the unknown plays in the situation posed; this arithmetic-algebraic characteristic allows the identification of the types of relationships that the student establishes in a problem (Vergnaud, 2020). Therefore, by analyzing the resolution of such problems, it is possible to identify which types of structures present difficulties for students. This insight is relevant for establishing potential intervention strategies in the students' learning process (Márquez et al., 2021). Furthermore, identifying these difficulties can help explain the obstacles students encounter when transitioning from arithmetic thinking with natural numbers to other sets, such as rational or real numbers. Inadequately developed implicit models of operations with natural numbers often lead students to extend these operations to other types of numbers, resulting in contradictions when solving problems (González et al., 2019).

Several investigations have shown that the type of problem presented influences the learning of mathematical content (Castañeda et al., 2019). Therefore, it is important to consider what kind of structure a student builds when solving a mathematical task (Bell et al., 1984) and to identify what kinds of connections they can establish between operations when modifying the numerical set (Empson & Levi, 2011; Sun, 2019; Van Hoof et al., 2022).

Moreover, some authors suggest that changing the problem structure poses significant difficulties Specifically, when incorporating division problems, students experience difficulties, especially when the divisor is greater than the dividend (Márquez et al., 2021). Although there is considerable interest in the study of problem solving, there is limited research demonstrating the variation in students' understanding of multiplication problems when the implicit algebraic structure is modified (Zorrilla et al., 2019). This study aims to show the influence that the type of multiplicative structure has on the success levels of fifth-grade students (10 to 12 years old) when solving multiplication problems with natural numbers.

This article explores problem solving for three types of multiplicative structures. In the first, isomorphism of measures problems, characterized by a proportion between two measure spaces (Vergnaud, 1983). Specifically, in this structure there are three types of problems, in primary basic education, according to Zorrilla et al. (2023) these are: (a) multiplication, where the unknown is the total quantity; (b) partitive division, where the unknown is the quantity per group; and (c) measurement division, where the unknown is the number of groups.

In the second structure, a single measurement space, distinguished by a correspondence is established between two quantities and a scalar operator. In this structure there are three types of problems, these are: (a) multiplication with comparison of a measurement as an unknown value; (b) division, where the incognita is a reference measurement; and (c) division with an unknown scalar (Ivars & Fernández, 2016).

Finally, in the third structure, there are the product of measures problems. Finally, in the third structure are the product of measures problems. Two types of problems can be identified: (a) multiplication, where the result of the multiplication is unknown but its factors are available; and (b) division, where one of the factors is unknown (Márquez et al., 2021). The way to differentiate each structure through the unknown present in each type of problem is presented in Table I (adapted from Ivars & Fernández, 2016).

Table I. Types of multiplicative structure problems.

Structure	Type	Incognita
Isomorphism of measures	Multiplication	Total objects
	Partitive division	Number of objects per group
	Measurement division	Number of groups
Single measurement space	Multiplication	Comparative quantity
	Division 1	Reference quantity
	Division 2	A scalar
Product of measures	Multiplication	Product size
	Division 3	An elementary measure

Materials and Methods

In this exploratory-descriptive study, 500 primary school students from five different schools in Colombia. The distribution of the participants is: Andean region (100), Caribbean region (100), Orinoco region (100), Pacific region (100) and Amazon region (100). According to the Colombian curricular guidelines, the study of multiplication begins in the 2nd grade of primary school (7 to 8 years old), increasing the number of digits involved progressively in the third and fourth grades. From this course onwards, algorithms with other numerical sets such as rational numbers, integers and then real numbers are also introduced.

For this reason, fourth grade students (10 to 12 years old) were selected, since it is a grade in which students are expected to master the multiplication algorithm. That is, it is expected that in this grade the algorithm as such is not an obstacle, which allows full attention to be paid to the resolution strategy in each problem. For data collection, a questionnaire was designed with eight problems (Table II): three measurement isomorphism (I), three single measurement space (SM) and two measurement product (PM) problems. The type of problem was varied according to the typology corresponding to each type of structure.

Table II. Problems presented in the questionnaire.

Structure	Type	Problem
Isomorphism of measures	Multiplication	In my house there are 7 rooms. If in each room there are 2 windows, how many windows are there in total?
	Partitive division	A teacher has 20 books that he wants to distribute equally among the 4 groups formed by his students. How many books should he give to each group?
	Measurement division	In a shopping mall, 40 video games were given away among the fans present. If each of the winners was given 4 video games, how many fans won video games?
Single measurement space	Multiplication	In my soccer class 8 balls were used, if my cousin's class used 4 times as many balls, how many balls were used in my cousin's class?
	Division 1	To paint my house this year we used 5 times as much paint as last year. If we used 15 gallons of paint this year, how many gallons of paint did we use last year?

	Division 2	If John has 6 toys and Mary has 24, how many times more toys does Mary have than John?
Product of measures	Multiplication	Mark has 3 shirts and 7 pants how many different ways can he combine them to dress?
	Division 3	Marcos has 60 ways to combine his shirts and pants to get dressed. If he has 12 shirts, how many pants does he have?

I Participants were given 90 minutes to complete the questionnaire individually in their regular classroom. The instructions given to the students were: (a) all answers must be accompanied by the solution process; and (b) no calculators or electronic devices could be used. To analyze the data obtained, each student response was taken and classified as correct (1) or incorrect (0). In addition, it is classified if the solution method used was correct (1) when following the structure corresponding to the problem or not (0) otherwise. Finally, the type of strategy used by each student is established.

Results and Discussion

The results are presented in three sections. The first shows the overall success level of the students. The second section presents the results related to the identification of the correct multiplicative structure. Finally, the third section presents the results obtained regarding the classification of the use of solving strategies, indicating whether they were correct or incorrect.

Global results

Table III shows the overall percentages of correct answers for each region and problem type. Students were most successful on the measurement isomorphism problems, followed by the single measurement space problems, while the measurement problems had a low percentage of correct answers.

That is, in general, students face greater difficulty in solving multiplication problems in which only two numbers are related to find an unknown. These types of problems require a greater understanding of the meaning of the operation, making it necessary to understand the function of each component of multiplication, otherwise the measurement isomorphism problems in which the data can be organized to apply the algorithm. multiplication more immediately.

Table III. Percentages of correct answers.

Structure	Type	Percentage				
		Andean region	Orinoco region	Caribbean region	Pacific region	Amazon region
Isomorphism of measures	Multiplication	60	45	44	39	36
	Partitive division	58	43	40	37	30
	Measurement division	61	41	41	40	35
Single measurement space	Multiplication	57	41	44	39	35
	Division 1	32	28	25	30	24
	Division 2	41	36	36	31	27
Product of measures	Multiplication	46	33	38	33	29
	Division 3	25	18	20	18	12

Additionally, there are some significant differences in the percentages of correctness by region, this can be explained by the fact of the strong cultural and socioeconomic differences of the regions. For example, while in the Andean region most schools have internet, one teacher for each grade, access to a feeding program for all students, in the Amazon region access to any technological resource is minimal, just one teacher It should guide the simultaneous learning process for all primary grades, etc. However, the important thing about the percentages is that they show that the trend, in terms of the type of structure that represents the least success, remains independent of the region taken.

Identification of multiplicative structure

Although the correct solution to a problem is an indicator of the level of understanding that a student may have about some mathematical object, an incorrect answer does not always indicate a total lack of understanding. More precisely, by requesting the solution process from students, the intention is to obtain as much information as possible about their way of understanding the structure of the problems.

For this reason, students' incorrect solutions were identified, in which the appropriate multiplicative structure was used. From this, the results presented in Table IV were found, where the percentage given is with respect to the total number of participating students in each region. For example, in the Andean region 60% of the students gave a correct solution to the problem, but there is another 15% who identified the correct structure to solve the problem, failing in the calculations, and the remaining 25% did not identify the appropriate structure.

Table IV. Percentages of correct structure.

Structure	Type	Percentage				
		Andean region	Orinoco region	Caribbean region	Pacific region	Amazon region
Isomorphism of measures	Multiplication	75	50	64	49	45
	Partitive division	61	45	41	39	33
	Measurement division	70	46	49	45	44
Single measurement space	Multiplication	62	47	51	49	49
	Division 1	32	28	25	30	24
	Division 2	41	36	40	38	29
Product of measures	Multiplication	58	39	43	39	43
	Division 3	25	18	20	18	12

From the results presented in Table 5, it is found that the trend of low successes in the multiplicative problems of Single measurement space and the product of measurements continues. However, it reveals an increase in the correct answers of all types of problems except those related to Division 1 in which the reference quantity is the unknown, nor in those of Division 2 in which one of the measurements is unknown. In other words, these two types of problems are found to represent the most profound difficulty in students' understanding.

To exemplify the data presented in this section, three solutions to the Division 1 problem (measurement as incognita) are presented in Figure 1. It can be seen that student one gives a correct solution, student two selects his strategy well but makes a mistake in identifying the number of times he should perform the operation, and student three makes a mistake throughout his analysis.

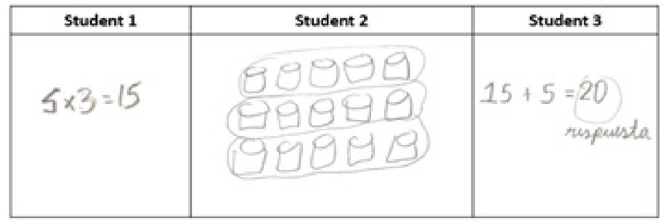

Figure 1. Different solutions to multiplication problem of the Division 1 type.

Problem solving strategies

To establish the strategies, each researcher separately analyzed the students' solutions and proposed a set of categories, which were then refined through a joint analysis. Finally, the strategies present in all the solutions to the problems were classified. This process generated 5 categories for correct strategies and another 3 categories for incorrect strategies. A strategy was considered correct if there was evidence that the solver recognized the multiplicative relationships between the quantities defining the situation.

The five correct strategies were: i) graphing (C1), ii) counting (C2), iii) use of multiplication tables (C3), iv) use of the algorithm (C4), and v) addition (C5). As for the incorrect ones, the following were found: i) Use of the inverse algorithm (I1), ii) Inappropriate operations (I2), and iii) Others (I3), where meaningless strategies and blank answers are grouped. Each of the strategies is exemplified below.

Graphic strategy (C1)

In this strategy students constructed graphical representations to represent relationships between quantities. More precisely, two types of relationships were found: groupings and apportionments. Examples of each of these graphical relationships are presented in Figure 2, in the case of grouping, the student represents the rooms and then groups quantities of windows until arriving at the answer. In the other case, the student creates the groups as circles and distributes in them dots that symbolize books.

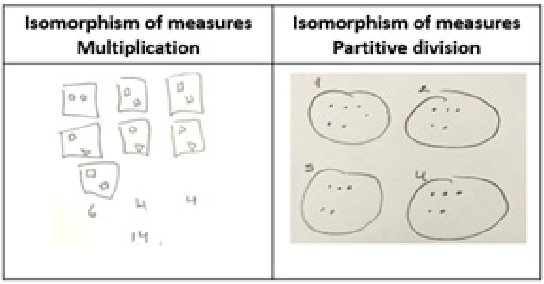

Figure 2. Examples of graphic strategy.

Counting strategy (C2)

This strategy was employed by students who resorted to adding the same quantity until they found the desired result. Figure 3 shows an example of how a student used it to solve a Single measurement space problem (Division 2). The student added 6 several times until he reached the target value of 24 toys, then counted the number of numbers he wrote down.

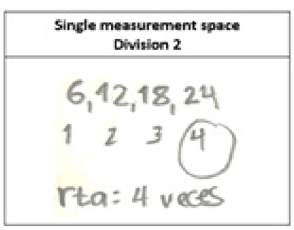

Figure 3. Examples of counting strategy.

Use of multiplication tables (C3)

Some students resorted to the construction of multiplication tables to identify the relationship posed in each problem. Figure 4 shows one of the solutions associated with this type of approach to the proposed situations, where it can be seen how a student reconstructs the multiplication table of the number 8 to give his answer.

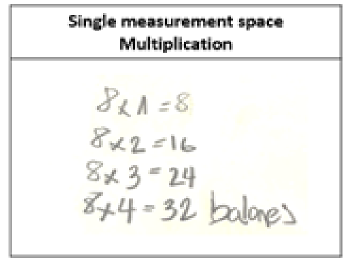

Figure 4. Example of use of multiplication tables.

Use of the algorithm (C4)

One of the strategies most used by the students who gave correct solutions to the problems was the use of the multiplication algorithm. Especially in the problems in which the presence of the values to be multiplied is made explicit, specifically those of multiplication corresponding to the isomorphism of measures and single measurement space. Figure 5 shows an example of using the multiplication algorithm to solve the Division 3 problem.

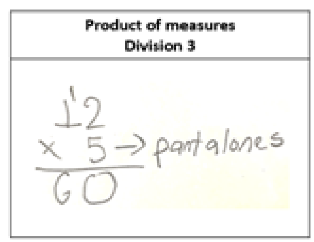

Figure 5. Example of the algorithm.

Addition (C5)

The last strategy that was properly employed was addition. In this strategy, students avoided direct multiplication and resorted to its interpretation as repeated addition to solve the problems. Figure 6 shows an example of the use of this strategy to solve a problem of Single measurement space problem, the student first takes the 2 and adds it until it exceeds 15, then tries with the 3 and finds the requested value.

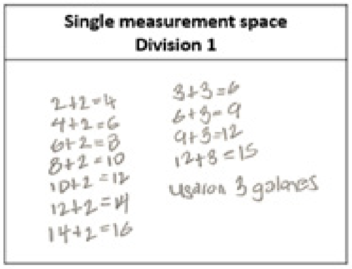

Figure 6. Example of use of addition.

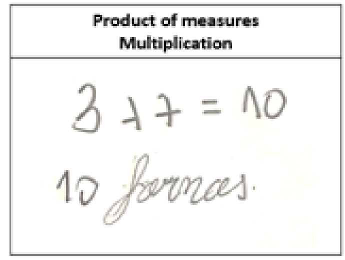

Figure 8. Example of inappropriate operations.

Use of the inverse algorithm (I1)

One of the most recurrent difficulties in the process of solving the problems was the attempt to apply the division algorithm. Especially in the problem of the product of measures associated with division, since there the students had to use a dividend and a divisor each of two digits, an action in which most of the students failed by misusing the algorithm. This is exemplified in Figure 7, which shows how a student, when wanting to divide 60 by 12, ends up dividing 6 by 2.

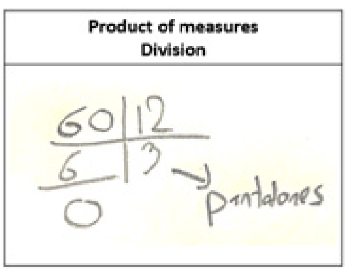

Figure 7. Example of use of addition.

Inappropriate operations (I2)

A recurring error identified was the strategy of adding or subtracting the values that appeared in the problem, that is, inappropriate operations were used for each type of problem. This occurred mostly in the problems in which positive quantities were to be multiplied and instead the students added them. An example of this difficulty is presented in Figure 8 corresponding to a solution to the Single measurement space (Multiplication) problem.

Others strategies (I3)

This category corresponds to the answers that did not have any type of procedure and in which there were values to which no meaning could be attributed within the problem. In addition, students who left the space for solving the problem blank were also counted. This category was found mostly in the product of measures problems as shown in Table V, which shows the overall percentages of presence of each of the strategies in the different types of multiplicative structure problems.

Table V. Problem solving strategies and structure.

Structure	Type	Percentage							
		C1	C2	C3	C4	C5	I1	I2	I3
Isomorphism of measures	Multiplication	2.8	7	0	28	7	0	45.2	10
	Partitive division	26	8	2.6	0	5	28	12	18.4
	Measurement division	6	16.6	0	0	21	27.4	22	7
Single measurement space	Multiplication	4.2	0	6	21	12	4	45	7.8
	Division 1	17.8	0	8	0	2	38	19.2	15
	Division 2	8.2	4	14	0	8	36.8	21	8
Product of measures	Multiplication	0	0	3	32.8	0	0	34.2	30
	Division 3	0	0	2.6	16	0	40	26	15.4

Let us remember the objective of this article: to examine the strategies used in the solution of multiplicative structure problems by students in the fourth grade of elementary school (from 10 to 12 years old). The results allow us to: (i) propose a classification of the problems according to the level of difficulty, and (ii) show which type of strategies tend to be more successful in the students' solutions.

These aspects are presented below in comparison with similar studies developed in other contexts.

Regarding the classification of the problems based on the level of difficulty, the following results are found, where 1 indicates the least difficulty found and 4 indicates the greatest difficulty:

Level 1. Isomorphism of measurements (Multiplication and Measurement division) and Single measurement space (Multiplication).

Level 2. Isomorphism of measures (Partitive division).

Level 3. Single measurement space (Division 2) and product of measurements (Multiplication).

Level 4. Single measurement space (Division 1).

Level 5. Product of measurements (Division 3).

In general, it has been found that division problems present greater difficulty, especially those in which the divisor and dividend are not simultaneously explicit. This result agrees with the findings of Zorrilla et al. (2023), who highlight that this difficulty is evident in all grades of primary basic education (from 6 to 12 years of age) in Spain. That is, this study concludes that division problems constitute an obstacle in the students' learning process, which leads us to reflect on the types of relationships that are typically established when solving this type of problems. In classrooms, students are often led to think that multiplication problems are totally disjoint from division problems, evading the transition between these two operations as a process of multiple connections and not simply as inverse operations (Levain et al., 2006).

Now, entering the specific case of explicit multiplication problems, it is found that in the three types of structure, students have a higher percentage of success in the association of values and unknowns because they find in them paradigmatic terms such as times or in each one, which are usually presented in Colombian classes as words associated only with multiplication. This greater degree of success in this type of problems is also found Ivars and Fernández (2016), who also highlight that problems in which the unknown is the compared quantity are usually easier for students than those in which those in those in which the referent quantity (scalar) is sought, as has been found in the present study with success percentages of 43.2% and 34.2% respectively.

To close this section, it is important to highlight that it is alarming that in all typologies of multiplicative structure problems a success rate of less than 50% was achieved. This shows a great deficit in the level of understanding of multiplication that students reach at the end of their primary school year. In this study, poor performance was found for Colombian students, but this situation was also found in the studies by Ivars and Fernández (2016), Zorrilla et al. (2023) and Cremades (2021) in Spain, as well as in the work of Márquez et al. (2019) in Chile, to mention a few countries.

Conclusions

In conclusion, it is observed that the learning of multiplication in Colombian primary basic education requires a profound review. The fact that more than 80% of the participating students cannot solve problems involving products of measurements that include division suggests that the work in the classroom is disconnecting two closely related operations, such as multiplication and division.

Regarding the types of multiplicative structure problems, the need for new teaching and learning strategies that allow students to develop a broad conceptual field on this mathematical object is highlighted. Only a global vision of the various situations in which multiplication acquires meaning can enable meaningful learning.

Finally, regarding students' resolution strategies, the analysis of the percentages of success in the relationship type of problem and strategy suggests an opportunity for the design and research of teaching and learning strategies. These proposals could be based on those strategies that have proven to be most successful, in order to develop activities that are based on them and that mobilize students towards a more general understanding of multiplication.

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