Perspectivas, 5 (2), Julio-Diciembre pp. 6-12, 2020, E ISSN 2590-9215

Revista Perspectivas Social Sciences Journal

Original Article https://doi.org/10.22463/25909215.2823

Application of Pólya's problem solving to the study of angles in fourth grade high school students

Aplicación de la resolución de problemas de Pólya en el estudio de ángulos en estudiantes de cuarto grado del nivel secundario

Rogel Rafael Rojas Bello[1], Esterlin Marysol del Rosario[2]

[1]Doctor en Ciencias Matemáticas, rogel.rojas@isfodosu.edu.do, ORCID: 0000-0002-9183-7572, Instituto Superior de Formación Docente Salomé Ureña, Santo Domingo, República Dominicana
[2]Licenciada en Educación Mención Matemáticas, e.marysoldelrosario@gmail.com, ORCID: 0000-0001-5583-3737, Instituto Superior de Formación Docente Salomé Ureña, República Dominicana

how to cite: R.R. Rojas Bello, E. Marysol del Rosario. "Application of Pólya's problem solving in the study of angles in fourth grade students". *Perspectivas,* vol. 5, no. 2, pp. 6-12, 2020.

Received: February 02, 2020; Approved: April 05, 2020.

ABSTRACT

In this research, a study is made of the effect of the application of Pólya's problem-solving heuristic on the development of geometric skills and its impact on academic performance. In the intervention, the topic of angles was developed, to a section of 25 fourth grade high school students in an educational center in the Municipality of Yamasá of the Monte Plata Province in the Dominican Republic. A quasi-experimental type design with a quantitative approach and correlational scope was used, with a non-probabilistic sample, with an intact group and a pre-test and post-test design. The results show that, through Pólya's problem-solving heuristic, students expanded their skills in geometry that greatly increased learning about angles. Corroborated by a growth in the average of the grades from 62.4% that was obtained in the pre-test, to 83.7% that was reached in the post-test, showing statistically that there are significant differences. It stands out that the Pólya heuristic was highly valued by the students under study, therefore, it is believed that it had a positive influence on the benefit of their learning about angles. In addition, the results of the post-test reveal that there is no relationship between the sex of the group under study and the scores obtained when the Pólya heuristic is applied.

Keywords:

Angles, grades, students, Geometry, Pólya, problems, Sex.

RESUMEN

En esta investigación se hace un estudio del efecto de la aplicación de la heurística de resolución de problemas de Pólya en el desarrollo de destrezas geométricas y su incidencia en el rendimiento académico. En la intervención se desarrolló el tema de ángulos, a una sección de 25 estudiantes de cuarto grado de secundaria en un centro educativo en el Municipio de Yamasá de la Provincia Monte Plata en República Dominicana. Es usó un diseño de tipo cuasiexperimental de enfoque cuantitativo y de alcance correlacional, con muestra no probabilística, con un grupo intacto y diseño de preprueba y postprueba. Los resultados muestran que, por medio de la heurística de resolución de problemas de Pólya, los estudiantes ampliaron habilidades en geometría que incrementaron en gran medida el aprendizaje en el tema de ángulos. Corroborado por un crecimiento en la media de las calificaciones de 62.4% que se obtuvo en la preprueba, a 83.7% que se alcanzó en la postprueba, mostrándose estadísticamente que existen diferencias significativas. Resalta, que la heurística de Pólya fue muy bien valorada por los estudiantes bajo estudio, por lo tanto, se cree influyó de manera positiva en beneficio de sus aprendizajes sobre el tema de ángulos. Además, los resultados que arrojan la postprueba revelan que no existe relación entre el sexo del grupo bajo estudio con las calificaciones obtenidas cuando es aplicada la heurística de Pólya.

Palabras clave:

Ángulos, calificaciones, estudiantes, Geometría, Pólya, problemas, sexo.

***Corresponding author.**
E-mail address: rogel.rojas@isfodosu.edu.do (Rogel Rafael Rojas Bello)

Introduction

Geometry is one of the oldest and most outstanding areas of mathematics, used for the service of society. It has a close relationship with various activities that take place in many branches of science, culture and recreational settings.

A good formation in Geometry guarantees in the student the capacity to understand the environment that surrounds him, prepares him for the inductive-deductive reasoning and develops his capacities to visualize, represent and solve an enormous amount of problems.

Problem solving, according to Aristizábal (2014, p. 15) defines it as: "the process through which we can recognize the signals that identify the presence of a difficulty, anomaly or hindrance to the normal development of a task, collect the necessary information to solve the detected problems and choose and implement the best solution alternatives, either individually or in a group", was developed by Pólya in 1945, year in which he started to be taken into account.

According to Pólya (1965), the resolution heuristic tries to somehow understand the procedure leading to the effective resolution of the problematic situations, particularly the relationships, associations and comparisons that the individual makes mentally during the whole resolution process. This heuristic is simplified into four stages: understanding the problem situation, establishing a certain plan, carrying out that plan, and finally, inspecting the solution obtained.

In this regard, Martinez (2018, p. 28) states that "problem solving is an inductive method of learning based on the search and discovery by students of answers, alternatives, relevant and timely solutions to the issues raised around a problem.

According to Barrantes, Balletbo and Fernandez (2014, p. 4): "in the methodology of problem solving, the role of the teacher is to choose situations and problems in order to awaken interest and encourage creative activity so that the student builds his or her own knowledge". Therefore, in the teaching-learning process of geometry, heuristic problem solving is based on the mastery of geometric concepts through activities closely related to a given context. That is, solving problematic situations of everyday life in which they are not always presented as refined situations.

Problem solving is not exclusive to the area of mathematics. In this regard, Jaime (2017, p.15) in his proposal of a methodological design based on Problem-Based Learning for the teaching and learning of chemistry in secondary schools reports that: "Among the changes found, there is a greater interest in reading a contextualized situation of the chemistry problem, more support among the members of the work team, distribution of functions among the participants of the work team, participation and personal satisfaction of each student when being part of a work group".

On the other hand, Gasco (2017, p. 54) in study carried out on the influence of the learning strategies in the learning of the mathematics to 565 students, of which 53% females and 47% males, belonging to educational centers of the public and private networks of the Basque Autonomous Community, concludes that the females tend to be more ordered and they manage the mathematical knowledge better than the males, besides requesting assistance more than the males when it is required. Although there is a similar employment in the stages of reproduction, elaboration, planning, following-up-regulation and the environment chosen to study.

According to a report presented by the Organization for Economic Cooperation and Development (2018, p. 7): "In the Dominican Republic, insufficient performance in mathematics has been demonstrated on multiple occasions, ranking among the lowest in evaluations such as the National Tests and the Program for International Student Assessment (PISA)". For this reason, it is necessary to introduce teaching and learning heuristics in the classroom that involve the students in the whole process, and make them the main actors of their own learning.

For the construction of Geometry, one of the most important concepts is that of angle, because it is fundamental for the study of objects, their relations between them and with the space or region they occupy. In addition, its properties are necessary in the study of all kinds of polygons.

In this order of ideas, Torres (2018, p. 28) in research carried out using a heuristic based on the resolution of problems of daily life, in the teaching of the subject of angles, in students of the second year of high school, the author considers that these have difficulties in the process of reading the problems and the conversion of radian units to sexagesimals and vice versa, they do not use the appropriate written mathematical language; furthermore, the author reported that they have problems in spatial location.

Martínez y Juárez (2019, p. 199), in research carried out on the resolution of daily life problems involving the topic of angles, in high school students whose ages range from 16 to 19 years old, the authors achieved that even when the students did not have solid knowledge about the stages of the Pólya heuristics in problem solving, they solved the problems following the stages not necessarily in a linear and consecutive way, but following the guiding questions of the first stages: understanding the problem and formulating a plan. And since students usually get used to doing a slight review of a problem and then try to solve it immediately, the problem statement did not explicitly exhibit a quick way of solution, so it was necessary to spend time understanding the problem, making some plan for the solution and analyzing that plan before carrying it out.

Therefore, it is necessary to search for efficient heuristics to be applied in the teaching-learning process on the subject of angles to high school students. Particularly, the problem solving heuristics of Pólya exhibit very striking characteristics for the study of the conceptual part, its properties and utilities of angles, since, due to the dynamics of Pólya's proposal, it motivates the student to learn and to transfer what he has learned to his daily life.

Materials and methods

The sample under study is non-probabilistic with an intact group and pre-test and post-test design. The focus of the research is quasi-experimental and correlational in scope, since it is intended to test the efficiency of the application of the Pollya method to students in a fourth grade section of high school and will use statistical analysis for measurements and numerical comparisons. In this regard, Hernández, Fernández and Batista (2014), when dealing with quantitative research, define it as: "A set of processes that is sequential and probative. Each stage precedes the next and we cannot jump or avoid steps. The order is rigorous, although of course, we can redefine some phases. It starts from an idea that is delimited and, once it is delimited, objectives and research questions are derived, the literature is reviewed and a framework or a theoretical perspective is constructed" (p. 4).

Research Context

The research was developed in the school year 2019-2020, in students of the subject Mathematics of fourth grade of the secondary level, because it is where the subject of angles in the subarea of Geometry is developed. This group under investigation is made up of 25 students, 14 are female (F) and 11 are male (M), with ages ranging from 15 to 19, where 16 students representing 64% are 16 years old.

Description of the Research Instrument

For the collection of the data the instrument used consisted of a questionnaire with three answer options, where the student could select only one. The instrument was developed by the researchers. Initially, it consisted of 13 questions, but these were reduced to nine during the external validation through the criteria of expert judges. The design of the questions responds to the competencies of the degree, which include the operations and transformations of angles from one system to another.

As Escobar and Cuervo (2008, p.29) argue: “Expert judgment is defined as an informed opinion of people with a background in the subject, who are recognized by others as qualified experts and who can provide information, evidence, judgments and assessments”. With the participation of six specialists in the field, the Binomial Test was applied, reaching significance indices of .01181 in the dimensions: coherence, relevance and clarity, which are less than .05 (< .05), which according to this criterion the instrument is valid.

For the internal consistency of the instrument, which according to Campos and Oviedo (2008, p. 832): “It refers to the degree to which the items, points or reagents that are part of a scale correlate with each other, the magnitude in which they measure the same construct”. The questionnaire was applied to nine 5th grade students at the secondary level, with similar characteristics to the sample. Kuder-Richardson’s test (KR-20) was applied, which according to Bójorquez and others (2013), quoted by Arévalo and Padilla (2016, p. 71): “This coefficient allows calculating reliability with only one application of the instrument, it does not require the design of parallel tests, and it is applicable only in instruments with dichotomous items, which can be coded with values of 1 and 0 (correct - incorrect, present - absent, in favor - against, etc.)”.

Therefore, the reliability indicator (KR-20) yielded an index of .819, which is a good reliability level for this test.

Results

To integrate the analysis, the grades were grouped into three intervals of variable amplitude, as shown in Table I. It was estimated that a grade lower than 70% is low, since it is the minimum passing grade in the Dominican educational system

Table 1: Ratings grouped by variable range intervals

Calificación	Cualidad
00 – 69	Baja
70 – 79	Regular
80 – 100	Buena

Pre-test phase

A pre-test was applied on the subject of angles, trying to detect the previous knowledge of the subjects under study. In relation to this diagnostic stage, Rojas-Bello (2020, p. 127) states that: “it serves to understand in what state the students’ previous knowledge is in general terms, in order to make timely decisions with a view to planning classes in such a way that they better respond to the students’ learning needs”.

Table II illustrates the results of this diagnostic phase (pre-test) for 25 students grouped by grade level, with 12 students representing 48% having a low score, nine students representing 36% having a fair score, and four students in the good category (16%).

Table II. PRE-TEST scores grouped by variable range intervals

Calificación	Frecuencia	Porcentaje
00 – 69	12	48
70 – 79	9	36
80 – 100	4	16
Total	25	100

Post-Test Phase

With the purpose that the students achieve the competences indicated in the course program, through the development of the Pólya heuristics, problems about angles were raised, which were worked on in groups and individually by the students, applying the Pólya heuristics in the whole process for its resolution. Later, through the post-test, the efficiency of the strategy was evaluated by selecting one of the three answers shown in the instrument.

In table III, the scores obtained in the post-test of the 25 students are grouped, observing that three students that represent 12% have a low score, eight students are in the category of regular (32%) and 14 students which represent 56% belong to the good category.

Table III. POST-TEST scores grouped by variable amplitude intervals

Calificación	Frecuencia	Porcentaje
00 – 69	12	48
70 – 79	9	36
80 – 100	4	16
Total	25	100

Pre-test and post-test comparison

Crossing the pre-test and post-test scores in Table IV shows that the frequencies of the low and good categories are very significant numerically in both tests. A better visualization of these percentage results is illustrated in Figure 1.

Table IV. PRE-TEST AND POST-TEST grades grouped by variable amplitude intervals

Calificación	Frecuencia de preprueba	Frecuencia de postprueba
00 – 69	12	3
70 – 79	9	8
80 – 100	4	14

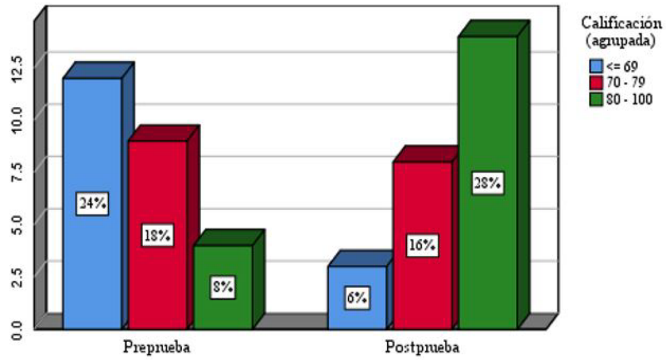

Figure 1. Frequencies of questionnaire scores: pre-test and post-test

Table V shows the descriptive statistics of the pre-test and post-test. It can be seen that the average corresponding to the pre-test (62.4) is numerically very different from the average corresponding to the post-test (83.7). In addition, the values of the respective deviations indicate that the scores in the post-test are closer to the average. However, it is theoretically demonstrated that there are differences.

Table V. Descriptive statistics of the pre-test and post-test

		Preprueba	Postprueba
N		25	25
Media		62.4	83.7
95% de intervalo de confianza para la media	Límite inferior	51.1	77.6
	Límite superior	73.6	89.8
Media recortada al 5%		64.3	84.8
Mediana		78	89
Varianza		746	216.2
Desviación		27.3	14.7
Mínimo		0	44
Máximo		89	100

Correlation analysis of pre-test and post-test

The use of parametric tests on pre-test and post-test data requires that they comply with normality. Therefore, applying the Shapiro-Wilk test with a confidence value of .05 (5%), using SPSS software version 25 for the calculations, a significance of P_value=.002 is obtained, which reveals that it is very small, that is: P_value<.05, that according to this test there is no homogeneity of the variances. Therefore, a non-parametric test is used, in this case the Wilcoxon test for related samples, resulting in a very small significance level (P_value=.007), which according to what is established for this test, there is a difference between the pre-test scores and the post-test scores. On the other hand, in Table V it can be seen that the 95% confidence intervals of the pre-test and the post-test do not coincide in any data, that according to this criterion there is a statistically significant difference between the pre-test grades and the post-test grades. To better visualize this situation, Figure 2 shows that there is no overlap between the two confidence intervals.

To study the correlation between the pre-test and post-test scores, we applied Spearman's non-parametric test, resulting in a significance P_value=.0003 and a correlation coefficient of approximately .5, which according to this statistical test there is a correlation with positive direction of moderate magnitude (mean) between both

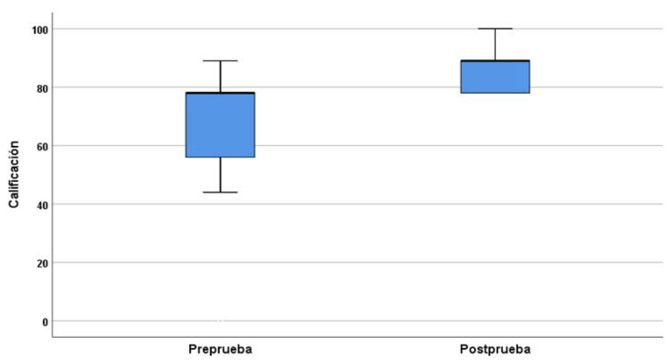

Figure 2. Comparison of means with 95% confidence interval.

Table VI. Results of grades versus sex in the post-test

		Sexo		
		F	M	total
Calificación	00 – 69	2	1	3
	70 – 79	4	4	8
	80 – 100	8	6	14
	total	14	11	25

Conclusions

Pólya's problem-solving heuristics in the development of geometric skills on the subject of angles in the 4th grade students at the secondary level proved to be efficient. This was evidenced by an increase in the average pre-test score of 62.4 to 82.7 in the post-test, resulting in significant differences on statistical tests. Also, it is verified that 88% of the students' grades in the post-test are in the category of regular or good, which contrasts with the results of the grades in the pre-test that resulted 52%.

A higher level of autonomy and control of learning was observed in the students under study, when the Pólya problem-solving heuristics were applied, which favored the high level of achievement of the specific competencies proposed for the fourth grade of secondary school, which is corroborated by the progress achieved through the post-test. In addition, Pólya's heuristic problem solving was highly valued by the students and is believed to have had a positive impact on the achievement of their learning on the subject of angles.

There is statistical evidence that Pólya's problem-solving heuristics, applied to the group of 25 fourth-grade students at the lower secondary level, did not influence the results of the grades by gender.

References

Arévalo, D & Padilla, C. (2016). Medición de la confiabilidad del aprendizaje del programa RStudio Mediante Alfa de Cronbach. Revista Politécnica, 37(1), 68-75. Recuperado de https://cutt.ly/xhrSpEM

Aristizábal, C. (2014). Fortalecimiento del proceso de comprensión de problemas matemáticos, a través del diseño y la implementación de un Material Educativo Computarizado (Tesis de Maestría), Universidad Nacional de Colombia-Sede Manizales. Recuperado de https://core.

ac.uk/download/pdf/77267033.pdf

Barrantes, M., Balletbo, I. & Fernández, M. (noviembre de 2014). Enseñar geometría en secundaria. I Congreso Iberoamericano de Ciencia, Tecnología, Innovación y Educación. Recuperado de https://cutt.ly/phrDdNE. Congreso llevado a cabo en Buenos Aires, Argentina.

Escobar, J. & Cuervo, A. (2008). Validez de contenido y juicio de expertos: una aproximación y su utilización. Avances en Medición, 6(1). 27-36. ISSN: 1692-0023.

Gasco, J. (2017). Diferencias en el uso de estrategias en el aprendizaje de las matemáticas en enseñanza secundaria según el sexo. Cuadernos de Investigación Educativa, 8(1), 47-59. Recuperado de https://dx.doi.org/10.18861/cied.2017.8.1.2638

Hernández, R., Fernández, C. & Baptista, P. (2014). (6ta Edición). Metodología de la Investigación. México: Mc Graw-Hill. ISBN: 978-1-4562-2396-0.

Jaimes, L. (2017). Propuesta metodológica para la enseñanza de la química en la Educación Media apoyada en el aprendizaje basado en problemas (APB). Perspectivas, 2(2), 6-16. Recuperado de https://doi.org/10.22463/25909215.1310

Martínez, G. & Juárez, E. (2019). La cubicación de madera como un problema geométrico real diseñado para promover el desarrollo de habilidades en la resolución de problemas. Acta Latinoamericana de Matemática Educativa, 32(1), pp. 191-199. Recuperado de http://funes.uniandes.edu.co/13982/1/Martinez2019La.pdf

Martínez, T. (2018). Influencia del método didáctico de resolución de problemas en el aprendizaje de Matemática I de los estudiantes del I ciclo, especialidad Matemática de la FAC-Universidad Nacional de Educación (Tesis de Maestría). Universidad Nacional de Educación, Lima, Perú. Recuperado de https://cutt.ly/phrSFVq

Organización para la Cooperación y el Desarrollo Económico. (2018). Recuperado de https://cutt.ly/ChrS0zE

Pólya, G. (1965). Cómo plantear y resolver problemas. México: Trillas. Recuperado de https://cutt.ly/jhtyQWG

Rojas-Bello, R. (2020). Introducción del GeoGebra en el proceso de enseñanza-aprendizaje de Geometría a docentes en formación. Revista Caribeña de Investigación Educativa (RECIE). 4(1). 124-134. Recuperado de https://doi.org/10.32541/recie.2020.v4i1.pp124-134

Torres, M. (2018). Midiendo Ángulos y Razón Trigonométrica, Aprendizaje Basado en la Resolución de Problemas de la Vida Cotidiana (Tesis de Maestría). Universidad Nacional de Educación, Ecuador. Recuperado de URI: http://repositorio.unae.edu.ec/handle/56000/894