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Underlying Factors in Students' Attitudes Towards Mathematics: A Factor Analysis and Its Educational Implications

Factores subyacentes en las actitudes de los estudiantes hacia las matemáticas: un análisis factorial y sus implicaciones educativas

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ABSTRACT

Keywords:

Attitudes Towards Mathematics, Exploratory Factorial Analysis, Confirmatory Factorial Analysis, Self-Perception, Teacher-Student Interaction, Mathematical Education.

Attitudes towards mathematics influence student performance and subsequent choices in scientific studies, necessitating their in-depth understanding. These attitudes can be influenced by various factors and are not limited solely to the understanding of mathematical concepts; they are also shaped by internal and external factors. Therefore, this study aims to analyse students' attitudes in relation to mathematics, as well as the existence of possible underlying factors of these attitudes. An exploratory investigation has been conducted using a sample of students from basic and secondary education institutions, which range from the fourth to the eleventh grade, in municipalities of the metropolitan area of Cúcuta, Colombia. An instrument consisting of 14 items was administered in printed format. With the participants' responses, an exploratory and confirmatory factorial analysis was performed with a good fit to a two-factor model. The first factor is linked to elements such as commitment and the perception of the teacher's role in mathematics learning. The second factor is associated with elements such as self-perception and perception of the mathematical context. These results are consistent with previous research, highlighting the multifaceted nature of attitudes towards this discipline. The analysis emphasizes that a positive relationship with the teacher and favourable self-perception are essential elements to promote positive attitudes. These aspects could be considered when designing future pedagogical interventions. The study provides a multidimensional perspective on attitudes towards mathematics, laying the groundwork for future research. It is essential to explore how these identified factors can influence and optimize students' educational experience in mathematics.

RESUMEN

Palabras clave:

Actitudes Hacia Las Matemáticas, Análisis Factorial Exploratorio, Análisis Factorial Confirmatorio, Autopercepción, Interacción Profesor-Estudiante, Educación Matemática.

Las actitudes hacia las matemáticas influyen en el rendimiento del alumnado y en elecciones posteriores de estudios científicos, lo que hace necesario su conocimiento con profundidad. Estas actitudes pueden estar influidas por diversos factores, y no se limitan únicamente a la comprensión de los conceptos matemáticos; también están moldeadas por factores internos y externos. Por ello, el presente estudio persigue analizar las actitudes de los estudiantes en relación con las matemáticas, así como la existencia de posibles factores subyacentes de estas actitudes. Para ello se ha realizado una investigación exploratoria en la que se ha empleado una muestra de estudiantes procedentes de instituciones educativas de educación básica y media, las cuales comprenden desde el cuarto hasta el undécimo grado, en municipios del área metropolitana de Cúcuta, Colombia. Se ha empleado un instrumento compuesto por 14 ítems que se ha administrado en formato impreso. Con las respuestas de los participantes se ha realizado un análisis factorial exploratorio y confirmatorio con buen ajuste a un modelo de dos factores. El primer factor aparece vinculado a elementos tales como el compromiso y la percepción del rol del docente en el aprendizaje de las matemáticas. El segundo factor se asocia con elementos tales como la autopercepción y percepción del contexto Matemático. Estos resultados concuerdan con investigaciones anteriores, resaltando la naturaleza multifacética de las actitudes hacia esta disciplina. El análisis subraya que una relación positiva con el docente y una autopercepción favorable son elementos esenciales para promover actitudes positivas. Estos aspectos podrían ser tenidos en cuenta al diseñar intervenciones pedagógicas futuras. El estudio proporciona una perspectiva multidimensional de las actitudes hacia las matemáticas, ofreciendo un fundamento para futuras investigaciones. Es esencial explorar cómo estos factores identificados pueden influir y optimizar la experiencia educativa en matemáticas de los estudiantes.

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Introduction

In the field of mathematics education in the Latin American context, there are no representative studies of the various existing environments. Therefore, research is being conducted to analyze the affective domain, pedagogical practice, and mathematical thinking processes in Colombian students. In this case, the study focuses on attitudes within that affective domain in mathematics.

Attitudes towards Mathematics

The current study is part of a broader research that examines the affective domain in relation to pedagogical practice and mathematical thinking processes. Fernández et al. (2018) identified a profound connection between the affective domain, which includes attitudes, beliefs, and emotions, and the pedagogical practices of pre-service and in-service teachers. Meanwhile, Prada et al. (2021; 2020) demonstrated how attitudes and beliefs towards mathematics can significantly impact academic performance. Prada et al. (2021) highlighted not only the need for reliable instruments to measure these affective and procedural dimensions in different countries, given the diverse realities in which students live, but also how the affective component can determine academic outcomes in mathematics. Furthermore, they emphasized the essential role of the teacher in forming constructive attitudes towards this discipline. Rincón-Álvarez et al. (2022) contributed to the development of the research by contextualizing it in Colombia and providing internal consistency values of a questionnaire on beliefs towards mathematics, representing a significant step in providing investigative tools. Prada et al. (2023) analyzed how various factors, including the affective domain, act as predictors of academic success in mathematics. These findings align with the approach of this study, which examines the multidimensionality of attitudes towards mathematics in relation to internal aspects of the student, such as their self-perception and confidence, and external aspects, including the role of the teacher in the student's learning experience. The mentioned research provides a robust framework that supports and expands our understanding of how attitudes can shape mathematical learning, contextualized in Colombia, laying the groundwork for future educational interventions in our region.

Mathematics transcends a simple academic discipline; it is considered a universal language that permeates various aspects of daily and professional life (Oneil, 2017). However, its teaching and learning present significant challenges (UNESCO, 2012). A fundamental problem lies in students' attitudes towards mathematics (Sanchal & Sharma, 2017). Many feels anxiety and apprehension towards this discipline, factors that can negatively impact their performance and professional development (Villamizar et al., 2020). Positive attitudes can enhance learning, while negative ones can represent significant barriers to achieving it (Romero–Bojórquez et al., 2014). Mathematics underpins disciplines and professions such as engineering and science (Godino, 2003). Attitudes towards it influence the choice of STEM (science, technology, engineering, and mathematics) careers (Avendaño & Magaña, 2017). A positive attitude enhances the choice of these areas essential for innovation (Marsh & Sharpe, 2020), while a negative attitude can have the opposite effect (Lane et al., 2022).

Students' Attitudes Towards Mathematics During Their Education in Primary and Secondary School

The transition from elementary to middle education marks a turning point in students' academic experience. In the mathematical realm, it is in primary elementary school where students have their first encounter with this discipline, forming their initial perceptions and attitudes towards it. However, as they advance to secondary and middle school, these perceptions not only solidify but also intensify, potentially adopting both positive and negative nuances.

Previous studies have shed light on this transition and the evolution of attitudes towards mathematics. Ashby (2009) points out that, in primary elementary, perseverance is crucial to tackle mathematics. Nonetheless, a negative attitude can hinder problem-solving abilities. Alarmingly, many students fail to identify the connection of mathematics with everyday situations, underscoring the need for greater contextualization in teaching. Motivation emerges as a key factor, especially for those with average or low academic performance. Moreover, it is at these levels that the first gender differences in confidence towards mathematical learning are observed, with girls tending to show less confidence in their abilities, despite not having lower performances.

The observations of Yara (2009) and Schenkel (2009) reinforce the importance of the teacher's role in shaping attitudes, as their influence can be decisive in the perception that students develop about mathematics. This influence becomes particularly relevant in secondary and middle school, as demonstrated by Hidalgo Maroto and Palacios (2006), who identify an increase in negative attitudes at this stage, particularly in compulsory secondary education in Spain (12-16 years).

However, the student's environment is not limited to the classroom. Núñez and others (2005) highlight the importance of the family environment as a fundamental pillar in mathematical learning, suggesting that the family can act as a support network or, conversely, contribute to the development of negative attitudes.

Reliable Instruments for Measuring Attitudes Towards Mathematics

Within the academic field, mathematics is recognized not only for its application and complexity but also for the affective responses it evokes in students. These responses, particularly attitudes towards mathematics, play a crucial role in effective teaching and learning. However, the lack of consensus on measuring these attitudes and the existence of multiple instruments with ambiguous theoretical foundations (Hannula, 2012) highlight the pressing need to evaluate and validate these instruments rigorously.

The research by Fernández Cézar and Aguirre Pérez (2010) provides an example of the attention given to this aspect. Their study on attitudes towards mathematics among students at the University of Castilla-La Mancha used the Auzmendi Attitudes towards Mathematics Scale. While this instrument addresses multiple affective dimensions, it is crucial to consider potential psychometric weaknesses, such as issues of validity, reliability, and response biases, especially when the scales of some questions are inverted.

Carmona-Marquez (2004) has emphasized that, without adequate validation and reliability of the instruments, the results obtained can be misleading, leading to potentially inefficient educational interventions. Likewise, despite their importance in mathematics education, the academic community has not yet reached a consensus on how to effectively measure these attitudes (León-Mantero et al., 2020; Ma & Kishor, 1997). Although different instruments have been proposed over time, some do not cover all relevant dimensions or focus only on specific aspects, such as math anxiety (Di Martino & Zan, 2011; Tapia & Marsh, 2004). In this context, it is essential that the academic community moves towards the adoption and validation of standardized and reliable measurement tools to accurately address attitudes towards mathematics.

In the referenced instruments for measuring attitudes, the use of exploratory and confirmatory factor analysis is missing. Along these lines, with the Auzmendi questionnaire, Fernández Cézar et al. (2016) evaluated the reliability through an exploratory and confirmatory factor analysis and discovered that the five-factor model was not suitable. They proposed a possible use of the questionnaire more focused on anxiety, obtaining a one-factor model. The instrument used in Prada Núñez et al. (2021) was created to analyze mathematical performance, highlighting the "Affective Domain towards Mathematics" based on questionnaires from Caballero et al. (2014) and Auzmendi (1992), and adding items from Emotions by Fernández et al. (2016), complemented with additional domains: "Mathematical Processes" (López et al., 2018) and "Pedagogical Practice". After a pilot with 292 students in Colombia and an Exploratory Factor Analysis, it was observed that some items needed to be reviewed due to discrepancies in their interpretation. For the Affective Domain, specifically in Beliefs towards Mathematics, it was proposed to increase items based on McLeod (1992), Callejo (1994), and Weiner (1986), but with caution not to compromise the effectiveness of the instrument. Regarding this instrument that has been used in various research in the Colombian context, there are still gaps in the research, and there is a clear need for instruments that consider all relevant dimensions.

Adaptation to Context

The adaptability and specificity of measurement instruments are essential for obtaining accurate and relevant results in research. As Zakariya (2018) points out, the influence of context is a fundamental aspect that should not be overlooked, and there is an imperative need to adjust the instruments to the specific contexts in which they will be applied. Rincón-Álvarez et al. (2022) emphasize the importance of contextualizing questionnaires, specifically when assessing beliefs about mathematics, highlighting the relevance of identifying and characterizing student beliefs in specific contexts such as the Colombian one, and even more so if they belong to contexts of social vulnerability.

Fernández-Cézar et al. (2020) explore how certain programs can affect attitudes towards mathematics and science in primary education, and in this work, the use of items that are particularly suitable for students in diverse contexts is promoted. This contextual approach is not only supported by previous research but also resonates with the direction and perspective of the current study. Understanding attitudes towards mathematics and their structure has consequences in education and other areas. This work seeks to provide a more detailed perspective, upon which more effective educational strategies can be created (Hannula, 2002; Ma & Kishor, 1997). Contributing to the validity and reliability of future research in this area is essential, as well as promoting equal opportunities in mathematical learning (OECD, 2016).

Attitudes towards mathematics play a crucial role in education and professional development. Understanding their nature, considering the specificity and particularities of each context (Zakariya, 2018), and how to measure them allows the development of more effective educational strategies, promoting more meaningful learning and greater equity in educational opportunities.

Given this context and understanding the significance of attitudes, the purpose of this study is to explore and understand the underlying structure of mathematical attitudes, selecting 14 items that reflect influential factors in such attitudes, based on the existing literature (Prada et al., 2021; Fernández-Cézar et al., 2016; Fernández-Cézar et al. 2020; Auzmendi, 1992).

Method

Research Approach and Design

This study adopts a quantitative approach with a non-experimental, cross-sectional correlational-causal design, suitable for exploring and analyzing the attitudes towards mathematics of elementary and middle school students. The choice of this design is due to its ability to gather a large amount of information in a short period of time, and its aptitude for factorial analysis (Creswell, 2014).

Sample and Context

A total of 1039 students were randomly selected from basic and secondary educational institutions, ranging from fourth to eleventh grade, in municipalities of the metropolitan area of Cúcuta, Colombia, enrolled between 2019 and 2020. The age distribution of the sample ranges from 16 to 18 years (M=17.3, SD=0.85), with 52.6% being female and 47.4% male. All members of the sample study mathematics as an integral part of their curriculum.

Ethical Procedures

Participants of legal age, that is, older than 18 years, provided their informed consent prior to participation. For minors, parental consent was obtained. The anonymity of the responses was guaranteed, ensuring their use exclusively for research purposes. No personal data were recorded, and the questionnaires were coded to preserve confidentiality.

Instrument

A 14-item questionnaire, derived from the 25 items proposed by Auzmendi (1992) as part of the instrument "Affective Domain in Mathematics and Initial Teacher Training" (Prada et al., 2021), was applied. This questionnaire focuses on key items related to attitudes towards mathematics, and its development was based on the extensive literature reviews conducted by Auzmendi, which include significant contributions from various authors, among them Aiken and Dreger (1961), Sandman (1974, 1980), Michaels and Forsyth (1977), Fennema and Sherman (1976), McConeghy (1985, 1987), Gairín (1987), and Tapia and Marsh (2004). To collect the participants' responses, a 5-point Likert scale was used, designed to assess the degree of agreement or disagreement with each item (Likert, 1932).

Table I. Attitude items from the questionnaire on the affective domain in mathematics and initial teacher training.

ATTITUDES	DESCRIPTION
Attitude 1	When I put effort into solving Mathematics exercises, I usually come up with the correct answer.
Attitude 2	Luck plays a role in successfully solving a Mathematics exercise.
Attitude 3	Mathematics becomes easier for me when the teacher uses various examples in class that relate it to real-life situations.
Attitude 4	When I notice the teacher's willingness to clarify doubts that arise during class, I feel more interested in Mathematics.
Attitude 5	Having good communication with the mathematics teacher sparks my interest in studying the subject.
Attitude 6	If the teacher explains with clarity and enthusiasm, it makes me like Mathematics.
Attitude 7	I feel committed to Mathematics when the teacher shows interest in my academic performance.
Attitude 8	I feel committed to Mathematics when the teacher values my effort in the subject.
Attitude 9	Having a family member who likes Mathematics makes me feel drawn to studying it.
Attitude 10	I feel different from others by the fact that I like Mathematics.
Attitude 11	The more I learn about Mathematics, the more competent I feel in society.
Attitude 12	I feel confident when solving Mathematics exercises.
Attitude 13	Mastering Mathematics will allow me to be successful in my further studies.
Attitude 14	Being good at Mathematics helps me perform well in other subjects.

Procedure

The students completed the questionnaire in printed format in their usual educational environment during class hours. This strategy ensured that the students were in a familiar environment and prevented them from feeling coerced into participating in the research.

Data Analysis

The statistical software SPSS version 25 and the IBM SPSS Amos 23 program were used for data analysis. Prior to the analysis, the data were reviewed for inconsistencies. If missing data were detected, the mean imputation method was employed (Schafer & Graham, 2002). A reliability analysis of the instrument was conducted using Cronbach's alpha, followed by exploratory and confirmatory factor analysis to discern the relationships between items.

Results

Reliability of the questionnaire

The Cronbach's alpha coefficient for the total sample was .910. A Cronbach's alpha coefficient above .7 is generally considered acceptable, suggesting that the questionnaire has good reliability and internal consistency (George & Mallery, 2003). A value of .910 indicates that the questionnaire items are consistently measuring the same underlying construct, in this case, attitudes towards mathematics. Therefore, this high Cronbach's alpha value suggests that the items in the mathematics attitudes questionnaire are consistent with each other in measuring this construct.

Factor Analysis

Factorial analysis is used to explore the underlying relationships among a set of observed variables and is based on certain assumptions. A key assumption is that the variables are correlated in the population, meaning there is some degree of relationship among them. To assess this assumption, two main tests were conducted: Bartlett's test of sphericity and the Kaiser-Meyer-Olkin (KMO) measure (Table II).

Table II. KMO and Bartlett's Test

KMO AND BARTLETT'S TEST		
SAMPLE		TOTAL
Kaiser-Meyer-Olkin Measure of Sampling Adequacy		.944
Bartlett's Test of Sphericity	Approx. Chi-square	8354.941
	df	91
	Sig.	.000

According to Table II, Bartlett's test of sphericity was significant ($p < .001$), indicating correlations between variables and suitability for factor analysis (Field, 2009). The Kaiser-Meyer-Olkin (KMO) measure (Kaiser, 1970; Hutcheson & Sofroniou, 1999) was .944, exceeding the threshold of .5, indicating adequacy for analysis. Thus, the tests suggest suitability for factor analysis.

Exploratory Factor Analysis

Despite the high internal consistency when considering all items as part of a single instrument, exploratory factor analysis (EFA) was conducted to identify the underlying structure of variables. EFA naturally indicated the existence of two factors (components), that is, two underlying variables, without forcing the extraction of a specific number of factors. The proportion of total variance explained by these principal components is presented in Table III.

Table III. Total explained variance using the principal components method

Total explained variance										
Sample	Component	Initial Eigenvalues			Extraction Sums of Squared Loadings			Extraction Sums of Squared Loadings		
			% %	%		%	%			%
		Total	Variance	Cumulative	Total	Variance	Cumulative	Total	Variance	Cumulative
Total	1	6,983	49,880	49,880	6,983	49,880	49,880	6,194	44,240	44,240
	2	1,546	11,042	60,922	1,546	11,042	60,922	2,335	16,681	60,922

The first component explains 49.880%, the second adds 11.042%, totaling 60.922% with two factors. Table IV shows the component (factor) matrix and the loadings of each item on each factor.

Table IV. Principal components matrix of attitudes towards mathematics

COMPONENT MATRIX		
	Total Component	
	1	2
Act1 [When I make an effort to solve math exercises, I usually come up with the correct answer.]	,715	
Act2 [Luck influences successfully solving a math exercise.]		,634
Act3 [Mathematics becomes easier for me when the teacher uses different examples in class that relate to everyday situations.]	,816	
Act4 [When I see the teacher willing to clarify doubts that arise during class, I become more interested in mathematics.]	,842	
Act5 [Having good communication with the math teacher sparks my interest in studying the subject.]	,834	
Act6 [If the teacher explains math clearly and joyfully, it makes me like mathematics.]	,868	
Act7 [I feel committed to mathematics when the teacher shows interest in my academic performance.]	,824	
Act8 [I feel committed to mathematics when the teacher values my efforts in the subject.]	,858	
Act9 [Having a family member who enjoys Mathematics, I feel drawn towards its study.]	,476	,525
Act10 [I feel different from others because I enjoy Mathematics.]		,772
Act11 [As I learn more Mathematics, it makes me feel competent in society.]	,486	,568
Act12 [I feel confident when solving Math exercises.]	,545	,428
Act13 [Mastering Mathematics will enable me to succeed in further studies.]	,645	,408
Act14 [Being good at Mathematics helps me perform well in other subjects.]	,507	,535

Factor loadings in Component 1 are high (0.634 - 0.868). This component has an internal consistency (Cronbach's alpha) of 0.910. Component 2, with factor loadings between 0.408 and 0.772, shows an internal consistency of 0.928. In summary, both factors exhibit high internal consistency and explain 60.9% of the instrument's variance.

Confirmatory Factor Analysis

Confirmatory Factor Analysis (CFA) assesses the fit of the proposed structure for the data being handled. The proposed models were evaluated using fit measures (Table V).

Table V. Goodness-of-fit measures of models

	Measures of absolute fit			measures of incremental fit			measures of parsimony fit			
Models	CMIN/DF	Chi-square	RMSEA	CFI	TLI	NFI	PRATIO	PCFI	PNFI	AIC
1	10.491	0.000	.096	.913	.896	.905	.835	.763	.756	883.323
2	5.733	0.000	.068	.958	.948	.949	.813	.779	.772	514.220
3	4.271	0.000	.056	.972	.964	.964	.780	.758	.752	399.220

The third model is the one that showed the best fit, as inferred from the values of the Chi-square coefficients, RMSEA, CFI, TLI, and NFI (Brown, 2006; Byrne, 2016; Hooper et al., 2008; Hu & Bentler, 1999) displayed in Table VII. The structure extracted from this CFA model is depicted in Figure 2.

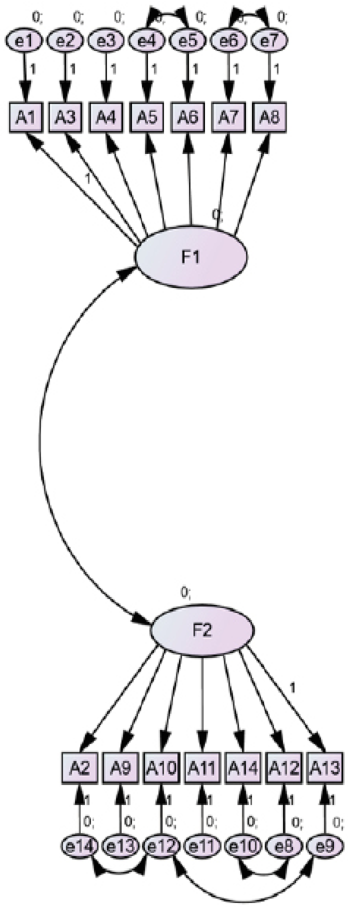

Figure 2. Factorial structure associated with Model 3 of CFA

The items in factors after CFA are shown in Table VI.

Table VI. Items for each factor

FACTOR	TOTAL SAMPLE
1	Act1, Act3, Act4, Act5, Act6, Act7, Act8,
2	Act2, Act9, Act10, Act11, Act12, Act13 Act14

Once the factors are detected, we could say, by observing the items that correspond to each of them, that the first factor, which explained 49.880% of the variance, focused on the commitment and perception of the teacher's role in Mathematics learning. The items grouped under this factor included statements such as 'When I make an effort to solve Mathematics exercises, I usually come up with the correct answer,' 'Mathematics becomes easier for me when the teacher uses different examples that allow me to relate them to everyday situations,' and 'I feel committed to Mathematics when the teacher values my effort in the subject.' This factor, which seems to reflect both the teacher's role in teaching and the student's effort and commitment, could be termed 'commitment and perception of the teacher's role' (Deci, et al., 1991; Hattie, 2009).

The second factor, which explained 11.042% of the variance, was related to aspects such as self-perception and perception of one's environment. The items grouped under this factor included statements such as 'Luck influences successfully solving a Mathematics exercise,' 'Having a family member who enjoys Mathematics attracts me to its study,' 'I feel different from others because I like Mathematics,' 'As I learn more Mathematics, it makes me feel competent in society,' and 'Mastering Mathematics will enable me to succeed in my further studies.' This factor seems to reflect how students perceive their relationship with mathematics in the broader context of their lives and how they perceive their competence and success in the subject. Therefore, it could be termed 'self-perception and perception of the mathematical context' (Bandura, 1997; Eccles, & Wigfield, 2002).

Analysis of the Relationship between Attitude Dimensions towards Mathematics

The study on students' attitudes towards mathematics focused on breaking down various dimensions associated with these attitudes. Once two main factors were identified, there arose the need to determine if there was any relationship between them. It was proposed that the items of each factor should be summed up and two distinct variables should be constructed, named "Factor1Act" and "Factor2Act". The correlation between these variables would be essential to understand if these dimensions act independently or share some underlying relationship. Although this process was mentioned, empirical evidence supporting this claim had not yet been presented. In order to provide this evidence and elucidate the possible connection, a correlation analysis based on the Pearson coefficient was conducted, suitable for large samples like the one in the present study. The results presented below detail the degree and significance of this correlation.

Correlation Analysis: A moderate and significant correlation was found between the two identified factors representing dimensions of attitudes towards mathematics, with a Pearson correlation coefficient of r = 0.541 and a p-value < 0.01 (see Table VII).

Table VII. Pearson correlation between factor1act and factor2act

		FACTOR1Act	FACTOR2Act
Factor1Act	Pearson Correlation	1	,541**
	Sig. (2-tailed)		,000
	N	1039	1039
Factor2Act	Pearson Correlation		
	Sig. (2-tailed)		
	N		
**. Correlation is significant at the 0.01 level (2-tailed).			

The value of r = 0.541 and a p-value < 0.05, with significance = 0.000, indicate that there is not enough evidence to accept the null hypothesis (H0: r = 0), and we accept the alternative hypothesis that there is a correlation between the two variables. However, correlation does not imply causation, as there may also be other unobserved factors and variables that could influence the results.

In summary, the provided results indicate that the instrument's reliability measure is excellent. The Cronbach's Alpha for the total sample is .910, suggesting a high degree of reliability of the instrument used. The Kaiser-Meyer-Olkin measure (KMO) for the total sample is .944, indicating excellent adequacy of the data for factor analysis. Additionally, Bartlett's test of sphericity showed statistical significance, allowing for continued factor analysis. Component 1 has factor loadings of 0.634 - 0.868 and a Cronbach's alpha of 0.910. Component 2 varies from 0.408 to 0.772 with an alpha of 0.928. The total variance explained by two factors is 60.9%, indicating a reasonable explanation of data dispersion with the selected factorial structure. The two identified factors relate to different questionnaire items. Each factor groups different attitudes towards mathematics, which can provide a deeper and differentiated understanding of how students perceive and relate to this discipline.

Regarding the possible relationship between the factors, it was determined by summing the items of each factor and constructing the variables "Factor1Act" and "Factor2Act", a moderate and significant correlation (r = 0.541, p < 0.01) between these dimensions was discovered, reaffirming that, although different, they share certain underlying characteristics. Confirmatory factor analysis (CFA) shows good fit indicators, with an RMSEA of .056 and CFI, TLI, and NFI all above .9 in the final model. This suggests that the two-factor model fits the data well. These results are essential for understanding students' attitudes towards mathematics and may form the basis for the development of pedagogical interventions aimed at improving perception and performance in this subject.

Discussion

The main purpose of this study was to examine students' attitudes towards mathematics, aiming to decipher the underlying factors that shape such attitudes. A quantitative approach was employed, adopting a non-experimental cross-sectional correlational-causal research design. This methodology was selected due to its effectiveness in data collection and its ability to facilitate factorial analysis, essential for unraveling the structure of mathematical attitudes (Aiken & Dreger, 1961; Sandman, 1974, 1980). The study sample comprised 1039 students from basic and secondary education levels in Cúcuta, Colombia. These participants, randomly selected, were representative of the educational context. Demographics focused on students aged between 16 and 18 years, with a balanced distribution across genders.

For data collection, a questionnaire consisting of 14 items was used, extracted from the attitude scale towards mathematics designed by Auzmendi (1992). This instrument stemmed from an extensive literature review on mathematical attitudes, considering seminal works such as those by Michaels and Forsyth (1977), Fennema and Sherman (1976), McConeghy (1985, 1987), and Gairín (1987). Additionally, other studies and findings were reviewed, indicating that, during basic and middle education levels, students begin to form their attitudes towards mathematics, which can significantly influence their performance and learning in this discipline.

According to this study, two primary factors emerge from our observations: 'teacher commitment and role perception', which explains 49.880% of the variance, and 'self-perception and perception of the mathematical context', which adds an additional 11.042%. This result supports our initial premise of the multidimensionality of attitudes towards mathematics (Deci et al., 1991; Hattie, 2009). The significant correlation observed between the factors also suggests complex interactions among these factors, which could even be influenced by other external variables. The negative correlation between internal and external aspects highlights an interaction between self-image and external influences, consistent with motivation theories (Bandura, 1997; Eccles & Wigfield, 2002). However, some absences of correlation suggest nuances not fully understood, prompting further research (Wigfield & Meece, 1988). Prada Núñez et al. (2021) also found that attitudes are grouped into two factors each, with at least 4 items or items in each.

While at first glance these two factors may appear different from those identified by Auzmendi, there are actually certain parallels and points of intersection. Anxiety in Auzmendi's scale may relate to elements of 'self-perception and perception of the mathematical context' in this study. Both dimensions touch upon aspects of students' self-image and emotional reactions towards mathematics. Auzmendi's enjoyment and motivation could be linked to 'engagement and perception of the teacher's role'. Enjoyment and motivation translate into active engagement with the subject, which may be influenced by the relationship with teachers. Utility, which highlights the perception of the value and applicability of mathematics in Auzmendi's scale, has elements that can be associated with both factors in this study. The perception of the practical value of mathematics can influence both student engagement and self-perception. Confidence overlaps with both dimensions of this study. Self-perception of ability and confidence can influence how students interact with their teachers and how they value mathematics in their lives.

Furthermore, the analysis reveals two factors that confirm and enrich the existing literature. The first factor, 'teacher engagement and perception of role', reflects interaction with teachers and mathematical problem-solving. Its composition, such as effort in exercises and teacher support, highlights the educational role in shaping positive attitudes towards mathematics (Hafızoglu & Yerdelen, 2019). For instance, Ashby (2009) points out in elementary education the importance of perseverance and everyday connections to enhance motivation in mathematics, while Yara (2009) emphasizes the role of teachers in fostering positive relationships with students. Schenkel (2009) also underscores that a positive attitude towards mathematics is correlated with better academic performance.

The second factor, 'self-perception and perception of the mathematical context,' composed of items revealing self-image and context, aligns with research indicating the relevance of social contexts and student self-assessment (Bandura, 1997; Gómez-Chacón, 2000). This influence is even more determinant in secondary and middle school, supported by Hidalgo, Maroto, and Palacios (2006). However, the student's environment is not merely academic; the family, as highlighted by Núñez and others (2005), plays an essential role in shaping attitudes towards mathematics.

The results suggest that teachers can foster positive mathematical attitudes by promoting communication and mathematical self-efficacy (Bandura et al., 2001; Deci et al., 1991). Considering the social environment is also crucial for shaping favorable attitudes (Leder & Forgasz, 2002). However, the instrument used, although reliable, has limitations. Although confirmatory factor analysis is robust and the sample is large and representative, contextual validity must be considered (Tavakol & Dennick, 2011). Validating the instrument across diverse groups is recommended.

For future research, delving into the interaction between factors and their influence on students' performance and interest in mathematics is crucial. Testing suggested pedagogical strategies and evaluating their effectiveness in the classroom would be beneficial. Additionally, replicating the study in various contexts or samples could provide more robust generalizations (Gioacchino et al., 2009). Overall, this research sheds light on the shaping of attitudes towards mathematics and guides towards a positive attitude in students, crucial for their educational success.

Conclusions

The study has unveiled the multidimensional nature of student attitudes towards mathematics, identifying two primary factors. The first is related to 'engagement and perception of the teacher's role'. The second factor is associated with 'self-perception and perception of the mathematical context'. Unlike many studies that might start with a hypothesis based on previous instruments like the Auzmendi questionnaire, this study did not propose an initial hypothesis, but aimed to explore whether there are underlying factors influencing students' attitudes towards mathematics. In this context, the results obtained validate this approach by providing a new perspective on the inherent complexity of attitudes towards mathematics in students. Although the specific names and structures of the factors may vary among different studies and instruments, there are consistent and recurring themes that emerge when addressing attitudes towards mathematics. The dimensions addressed by Auzmendi (1992), such as anxiety, enjoyment, usefulness, motivation, and confidence, are underlying themes, as are 'engagement and perception of the teacher's role' and 'self-perception and perception of the mathematical context' from this study, which are crucial for understanding and improving student attitudes.

Although a moderate correlation was identified between the two dimensions representing attitudes towards mathematics, it is evident that, despite being distinct, these dimensions share underlying characteristics or influences. However, this does not diminish the uniqueness of each factor; on the contrary, it reinforces the idea that each offers a unique perspective on the attitude towards mathematics. This uniqueness underscores the importance of conducting individualized assessments and interventions. In this context, it becomes crucial to delve deeper into both the practical and theoretical understanding of this relationship, and it is recommended to investigate the underlying variables that might be influencing this correlation. Despite the findings, the relevance of using tools that focus on the different dimensions of the attitude towards mathematics remains current, with the aim of ensuring a comprehensive and holistic assessment of students. Moreover, it is worth mentioning that, in response to previous observations, concrete statistical results were incorporated into the study, which strengthens the validity and reliability of our findings.

The visualizations and data analysis applied in our research provided a clear and intuitive representation of the emerging patterns in the data. This tool proved effective for understanding and communicating the results obtained. Additionally, although we did not directly address the differences in attitudes among different age groups and educational levels, the two factors identified lay a solid foundation for future research exploring these disparities.

The two factors found could be shaped by age and educational level, urging the consideration of these factors when developing educational strategies. Altogether, the study enriches the understanding of attitudes towards mathematics, providing two relevant factors for educators and others involved in improving students' mathematical experience. These results indicate the need to consider various dimensions and highlight the importance of future research to unravel the interactions and possibilities for positive influence.

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