



Estimación del Factor de Intensidad de Esfuerzo en una Probeta Wedge Splitting Bajo Carga Estática Mediante el Método de Elementos Finitos

Estimation of the Stress Intensity Factor in a Wedge Splitting Test Under Static Load Using the Finite Elements Method

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ABSTRACT

Keywords:

Stress intensity factor, Lineal elastic fracture mechanics, Finite element method, Wedge Splitting test.

The stress intensity factor (SIF) is a parameter to define the stress field to which a cracked body is loaded under the guidelines of linear elastic fracture mechanics (LEFM). The value of the stress intensity factor SIF is function of the applied stress, crack length and the general geometry of the specimen. There are several experimental methods to calculate this stress intensity factor, among which the Wedge Splitting Test (WST) stands out. In the present work, the stress intensity factor for a WST specimen of concrete is calculated using the finite element technique maintaining a constant load and making variations in the crack length and the geometry. Then the results are compared with an experimental correlation taken from the literature.

RESUMEN

Palabras clave:

Factor de intensidad de esfuerzos, Mecánica de fractura lineal elástica, Método de elementos finitos, Wedge Splitting Test.

El factor de intensidad de esfuerzos (SIF) es un parámetro suficiente para definir el campo de esfuerzos al cual está sujeto un cuerpo agrietado bajo los lineamientos de la mecánica de fractura lineal elástica (LEFM). El valor del factor de intensidad de esfuerzos, SIF, es función del esfuerzo aplicado, la longitud de la grieta y la geometría general de la probeta. Existen varios métodos experimentales para calcular este factor de intensidad de esfuerzos, entre los que se destaca el Wedge Splitting Test (WST). En el presente trabajo se calcula el factor de intensidad de esfuerzos para una probeta de WST de concreto sujeta a carga estática mediante la técnica de elementos finitos, manteniendo una carga constante y realizando variaciones en la longitud de la grieta y la geometría del cuerpo. El estudio se realiza con la finalidad de obtener una relación gráfica entre el factor de intensidad de esfuerzo y las diferentes variables en el ensayo. Los resultados obtenidos se comparan con una correlación numérica tomada de la literatura en donde se constatan resultados satisfactorios para la simulación.

Introduction

In order to perform a fracture mechanics design analysis, firstly, it is necessary to determine the fracture toughness of the material. Several tests have been developed over the years to determine this parameter, ASTM D7313 [1] uses the circular compact tension specimen for concrete, and the 3 point (point bending) specimen has also been used to obtain this property in high strength concrete [2]. On the other hand, the wedge splitting test [3] was introduced as a new method in 1990 with the purpose of determining the fracture toughness (K_{IC}) of the tested specimen material, as well as the specific energy required to produce the failure of the specimen. The test produces the fracture of the specimen by controlling the force applied symmetrically to the element using wedges and rollers (Figure 1). The specimen is placed on a fixed support while the load is applied. Two variables are measured during the test, the

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horizontally applied force (F_H) and the crack face displacement, better known as crack tip displacement (CTOD). Once the fracture occurs, the area under the graph F_H vs CTOD is calculated, this parameter is nothing more than the energy necessary to produce the fracture of the body. The amount previously calculated, divided by the area of the resistant zone, translated into the specific fracture energy (G). Several authors have used the WTS to determine the fracture toughness of concrete [4] [5], recognizing a great advantage of working with this specimen due to its manufacture and the control over controlled crack growth [Reference]. In this research, the real conditions of load and displacement of the test are recreated using ANSYS workbench 2019 software [6] to obtain a graphical relationship of great help at the time of executing the experimental test.

Methodology

The simulation performed uses the finite element method to recreate the WST test conditions. The results obtained are compared with results from the literature for another simulation performed where an expression is derived for the calculation of the stress intensity factor as a function of several dimensionless parameters, these values are taken as theoretical to calculate the percentage error between the two studies.

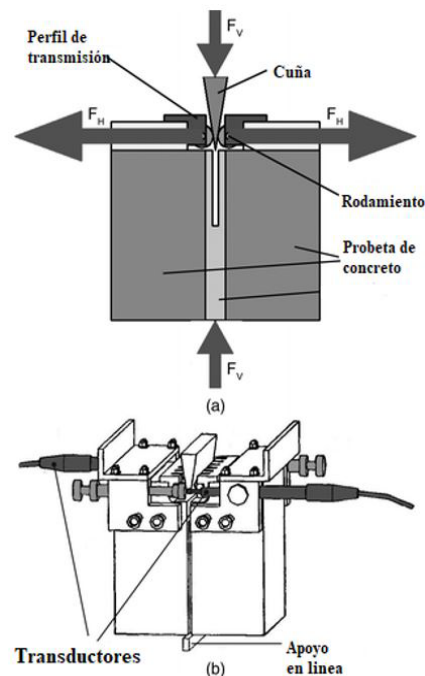


Figure 1. Wedge Splitting Test. a) Diagram of forces and supports, b) actual assembly.

Both in the test and in the simulation the influence of the vertical force subject to the angle of the wedge is disregarded, usually between 10 and 15°. During the test, the force is transmitted from the wedge to the specimen by means of 2 steel sections, these sections have 5 times the modulus of elasticity of the tested body, which mean, the concrete. The test is performed with this configuration in order to disregard the deformations in the steel during the application of the load (see equation 1).

$$\frac{E_{acero}}{E_{concreto}} = 5 \quad (1)$$

Where:

E_{acero} [MPa] corresponds to the modulus of elasticity of steel and $E_{concreto}$ [MPa] corresponds to the modulus of elasticity of concrete.

Due to the loading conditions, the specimen is subjected, this is subjected to a bending effort, allowing the presence of modes II and III to be neglected. Subjecting the specimen to loading mode I allows the specimen to fracture symmetrically in a direction perpendicular to the applied load, that mean, almost vertically. Since this test is mainly performed for brittle materials, the fundamentals of linear elastic fracture mechanics can be applied [7] [9].

The present study contemplates a three-dimensional geometry so that the variation of the stress intensity factor along the crack front can be analyzed. Figure 2 shows the specimen and all possible parameters to be evaluated.

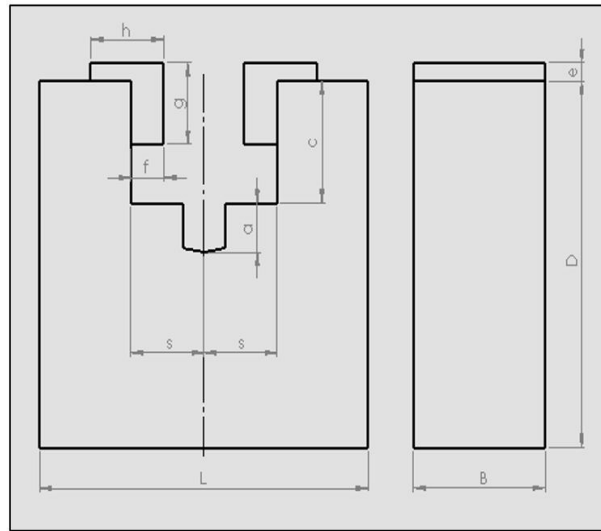


Figure 2. Geometry of the specimen.

The parameters that will remain fixed during the simulation and their values are:

$$\begin{aligned} c &= 30 \text{ mm} \\ p &= 6,5 \text{ mm} \\ h &= g = 20 \text{ mm} \\ s &= 20 \text{ mm} \\ f &= 9 \text{ mm} \\ e &= 4,5 \text{ mm} \end{aligned}$$

In this work a horizontal force F_H of 1000 N is used, values of the geometric condition $\frac{d}{c}$ equal to 3, 4,5,6,7 and 8 are treated. For each of these values, the dimensionless parameter $\frac{a}{d-c}$ is varied from 0.2 to 0.6 in order to see the influence of the variation of the crack length a on stress intensity factor.

The theoretical value of the stress intensity factor is calculated using the following correlation given by Guinea and Planas [10].

$$\frac{K_I * B * \sqrt{c}}{F} = 3,975 * \left[\frac{\frac{d-p}{c-c}}{\left[\left(\frac{d}{c} - 1 \right) * \left[1 - \frac{a}{d-c} \right] \right]^{1,5}} \right] \quad (2)$$

Where:

K_I [$MPa\sqrt{m}$] is the stress intensity factor for loading mode I, B [m] is the specimen thickness, a [m] is the crack size, F is the applied force, and c, d and p [m] are geometric dimensions of the specimen.

It can be seen that the above expression is expressed as a function of the specimen geometry and the applied load. A large number of expressions have been proposed to determine the stress intensity factor in similar tests [11] [12] [12] [13] [14]. In order to show the procedure performed, the parameters and tools used in the simulation are presented step by step:

A CAD modeling of the specimen in ANSYS Design Modeler.

B Configuration for a study under static loading using the static structural module.

C Assignment of the mechanical properties for the materials, keep in mind that they are modeled as isotropic homogeneous: $E_{steel} = 200GPa$, $\nu_{steel} = 0,3$, $E_{concrete} = 40 GPa$ y $\nu_{concrete} = 0,15$

D Assignment of contact conditions between the steel sections and the specimen, working with a fixed or bonded contact.

E Creation of the coordinate system at the crack tip, necessary for the calculation of the J-integral.

F Configuration and sizing of the tetrahedral finite element mesh in the structure, with a refinement in the contact zone and in the stress zone. A refinement of 2 times the element size is used in the zone near the crack tip (See Figure 3).

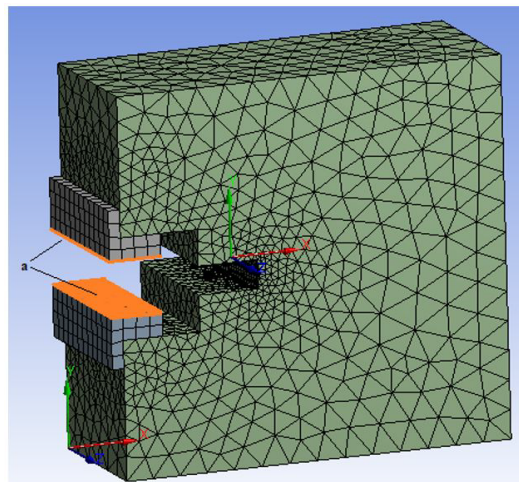


Figure 3. General mesh. a) Force F application surfaces.

G. Configuration of the fracture parameters in the test, a preformed crack with known length is programmed according to the case of study, for this crack size the stress intensity factors are subsequently calculated.

H. Definition of boundary conditions, loads and displacements. Two normal forces are implemented to the inner faces of steel profiles, these forces have equal magnitude (1000 N) and opposite direction as shown in Figure 3. To simulate the support at the bottom of the specimen, a movement constraint is implemented at the central nodes of the bottom face along the entire width (B) of the body.

I. Generation of results for the stress intensity factor by computing the J integral given by the following expression:

$$J = \int_S (W n_1 - t_j u_{j,1}) dS \quad (3)$$

Where:

W is the strain energy density, n_1 is the normal to the contour in direction 1, t_j is the strain and $u_{j,1}$ is the derivative of the displacement with respect to direction 1. Note that direction 1 is the x-axis. Once the J-integral is evaluated for any contour S in the counterclockwise direction, the stress intensity factor can be calculated with the following relation for isotropic homogeneous materials [15]:

$$J = \frac{1}{E} K_I^2 \quad (4)$$

Results

The SIF results for the K_{I} loading mode are expressed as a function of specimen depth in Figure 4, the software evaluates the stress intensity factor by means of the J integral [16]. The value of the J integral is independent of the integration path [17], in order to verify the influence of the worked mesh and the contour used in the SIF evaluation it is necessary to evaluate J for several contours around the preformed crack. Figure 4 presents the evaluation of 8 contours around the crack tip, where it can be observed that the results for contours 3 onwards present the same value for the stress intensity factor, this value, invariant with the contours, is the one taken as a result of the different simulations for the variations in the geometry of the body.

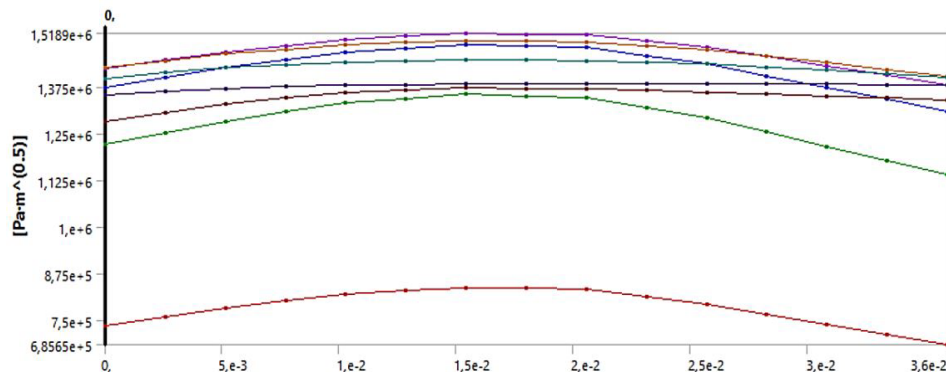


Figure 4. Stress intensity factor K_I at the crack face.

The figure shows the variation of the stress intensity factor with the width of the specimen (See Figure 4), this is due to the fact that there are two states present in the body: a flat deformation state in the central part of the specimen and a flat stress state in the lateral zones. According to the methodology, the different simulations are programmed for the values of $\frac{a}{d-c}$ and F and the following results are obtained.

Table I. Comparison of results for d/c=3.

L/d= 0,5 d/c=3 F=1000 N				
$\frac{a}{d-c}$	a[mm]	K_I TEÓRICO[Pa*m ^{0,5}]	K_I SIM[Pa*m ^{0,5}]	ERROR [%]
0,2	12	8,767E+05	8,901E+05	1,530
0,3	18	1,071E+06	1,085E+06	1,269
0,4	24	1,350E+06	1,356E+06	0,430
0,5	30	1,774E+06	1,807E+06	1,855
0,6	36	2,480E+06	2,502E+06	0,903

Table II. Comparison of results for d/c=5

L/d= 0,5 d/c=5 F=1000 N				
$\frac{a}{d-c}$	a[mm]	K_I TEÓRICO[Pa*m ^{0,5}]	K_I SIM[Pa*m ^{0,5}]	ERROR [%]
0,2	24	3,196E+05	3,235E+05	1,202
0,3	36	3,905E+05	3,976E+05	1,810
0,4	48	4,921E+05	5,037E+05	2,354
0,5	60	6,469E+05	6,581E+05	1,745
0,6	72	9,040E+05	9,074E+05	0,377

Table III. Comparison of results for d/c=7.

L/d= 0,5 d/c=7 F=1000 N				
$\frac{a}{d-c}$	a[mm]	K_I TEÓRICO[Pa*m ^{0,5}]	K_I SIM[Pa*m ^{0,5}]	ERROR [%]
0,2	36	1,762E+05	1,781E+05	1,062
0,3	54	2,153E+05	2,186E+05	1,537
0,4	72	2,713E+05	2,741E+05	1,033
0,5	90	3,567E+05	3,656E+05	2,511
0,6	108	4,985E+05	5,080E+05	1,916

These can be seen graphically in Figure 5.

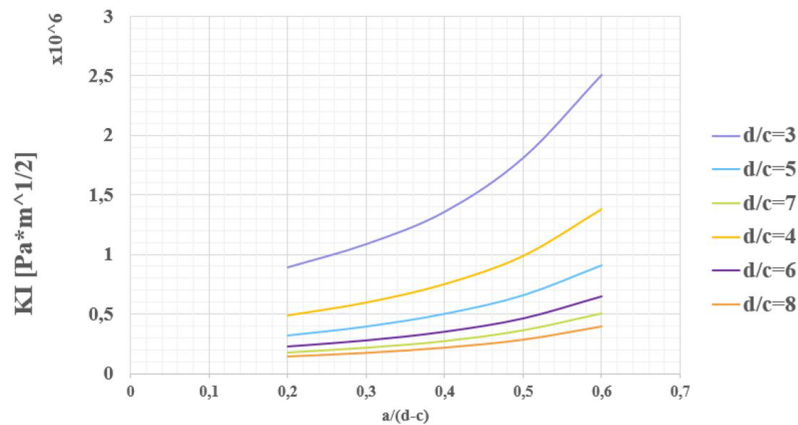


Figure 5. Results for the SIF.

In order to make the results more practical, the value of the stress intensity factor is plotted dimensionless on the ordinate axis (Figure 6).

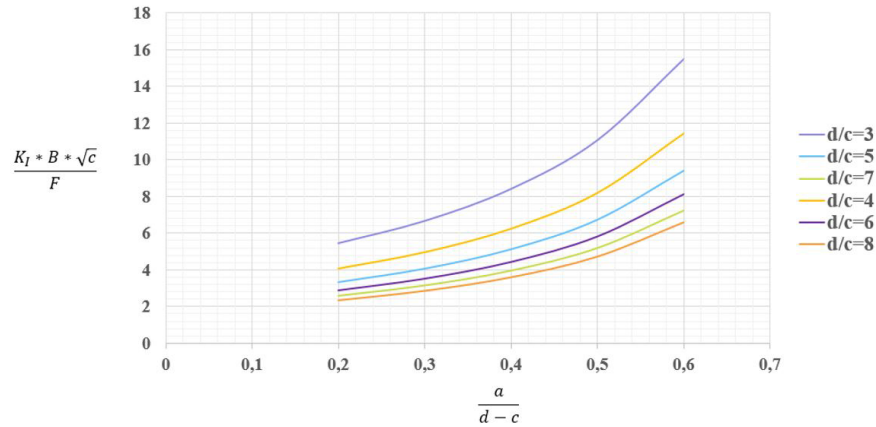


Figure 6. Results obtained in a dimensionless manner.

When working with the finite element method, it is necessary to perform a sensitivity analysis in order to verify that the results obtained converge to the desired solution, in another words, to find the minimum number of elements to be used, where the results remain stable. To carry out the analysis, variations are made in the size of the elements at the crack tip (See Figure 7), it is observed that the stress intensity factor converges to a specific value for each case of study. The simulated results are presented in Figure 8 as a function of the number of elements at the crack front.

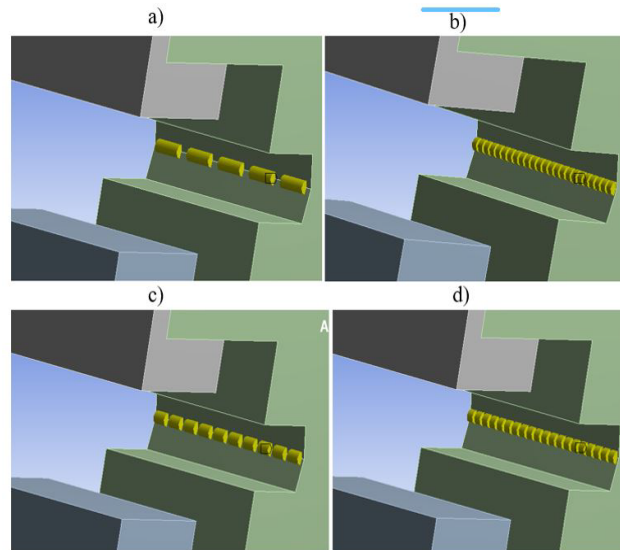


Figure 7. Variation of the number of elements in the crack front for sensitivity analysis. a) 5 elements, b) 24 elements, c) 10 elements, d) 20 elements.

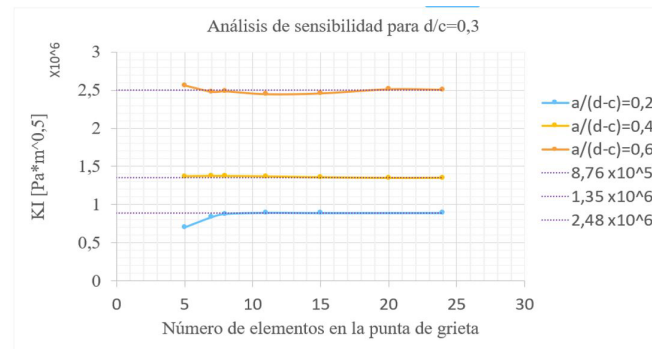


Figure 8. Sensitivity analysis for d/c=3.

After a minimum number of elements in the crack front the value of the stress intensity factor remains constant for each specific case, this convergence value was taken to compare it with the theoretical results of the experimental correlations of Guinea and Planas [10].

Conclusions

- With the obtained graphs it is possible to evaluate the critical stress intensity factor (fracture toughness) for a test performed if its geometry and the force at the moment of failure are known. Once the fracture toughness value of the material to be worked on is known, the fracture mechanics design process can be carried out by comparing the applied stress intensity factor with the fracture toughness, in another words using the failure criterion for static loads.
- The experimental analytical correlation of Guinea and Planas [10] is validated for the Wedge splitting test in concrete.
- Figures 5 and 6 show a strong dependence with specimen dimensions and crack size, so it can be stated that the relationship with the stress intensity factor is inverse with geometric dimensions and direct with crack size, the same behavior observed by Guan [18].
- The sensitivity analysis was performed for all simulated tests, therefore, it is corroborated that the presented results do not depend on the mesh refinement after implementing 10 or more elements at the crack front.

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