



Incidence of the orientation of the reinforcing fibers in a carbon/epoxy composite shaft, for torsional loads and rotational speed

Incidencia de la orientación de las fibras de refuerzo en un eje compuesto de carbono/epoxi, ante cargas torsionales y la velocidad de giro

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ABSTRACT

Keywords:

Composites,
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Critical speed, Ply.

The study carried out is based on the analysis of the behavior of the angle-ply of a composite shaft, in terms of its resistance to torsional loads and speeds of rotation. For reference, a composite shaft of carbon fiber and an epoxy polymer matrix is optimally designed. The orientations of the laminas in the laminate were selected to ensure their manufacture by the filaments winding method. For the specific case of the study, it can be concluded that the pairs of laminas with angles of 45° and -45° optimally satisfy the design parameters of the shaft. The orientations at 0° and 90°, are useful when the critical speed and torsional buckling are more restrictive.

RESUMEN

Palabras clave:

Materiales
compuestos, Eje
compuesto, Fibras
de refuerzo,
Velocidad crítica,
Apilado.

El estudio realizado se basa en el análisis del comportamiento de las orientaciones del laminado de un eje compuesto, en cuanto a su resistencia ante cargas torsionales y velocidades de giro. Como referencia se diseña óptimamente un eje compuesto de fibra de carbono y matriz polimérica epóxica. Las orientaciones de las láminas en el apilado se seleccionaron para garantizar su fabricación por el método de bobinado de filamentos. Para el caso específico de estudio se puede concluir que los pares de láminas con ángulos de 45° y -45° satisfacen de forma óptima los parámetros de diseño del eje. Las orientaciones a 0° y 90°, son útiles cuando la velocidad crítica y el pandeo torsional son más restrictivas.

Introduction

Composite materials today have become one of the main alternatives to conventional materials [1]. Characteristics such as its high adaptability to the most demanding design requirements, and its excellent mechanical and physical properties [2], provide engineers with versatility when developing their designs. One of the main reasons for its massive use today lies in the ability to provide high strength and rigidity with low weight, thus generating improvements in energy efficiency and significantly affecting a reduction in manufacturing and operating costs [3].

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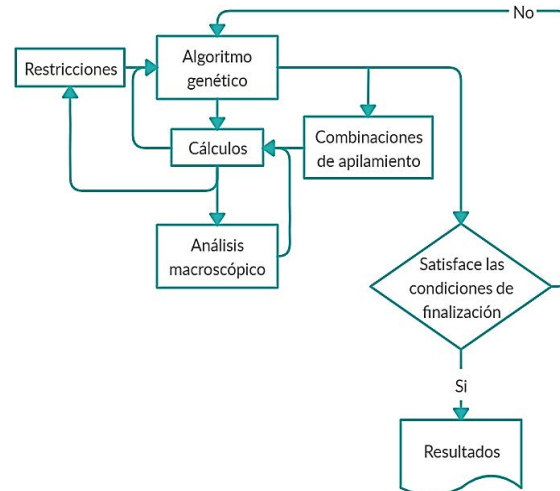


Figure 1. Flow diagram of the optimization algorithm used
Source: self-made

Optimal design

To analyze the incidence of the orientation of the reinforcing fibers in a composite shaft, the optimal design of a tubular shaft was carried out, according to the following considerations:

- Shaft length = 0.7 [m]
- Maximum torque = 500 [N · m]
- Maximum speed = 5000 [rpm]
- Maximum diameter = 50 [mm]
- Safety factor = 3
- There are no axial loads
- Manufacture by winding of filaments

The design was made with a composite material of carbon fiber and epoxy resin polymer matrix. The properties of the material [9] are shown in table 1.

Table 1. Mechanical properties of carbon fiber and epoxy resin composite.

Elastic modules	
Longitudinal E1	130 [GPa]
Transversal E2	7.6 [GPa]
Mod. shear G12	7.17 [GPa]
Coeff. by Poisson V12	0.28
Ultimate efforts	
Long. traction σ_{1t}	1371 [MPa]
Long. compression σ_{1c}	1317 [MPa]
Trans. traction σ_{2t}	40 [MPa]
Trans. Compression σ_{2c}	246 [MPa]
Couthing τ_{12}	68 [MPa]

Source: Self-made.

Optimization algorithm

The optimal design of the axis was carried out using a genetic algorithm, implemented from the “Global Optimization Toolbox” complement of the MATLAB work environment. The objective function to be minimized corresponds to the weight (W) of the axis and is given by:

$$W = L \cdot \pi (r_o^2 - r_i^2) \cdot \rho \cdot g \quad (1)$$

Where L represents the length of the axis, ρ the density of the composite material and g the gravity. The objective function is delimited by the following variables:

- The outer (r_o) an inner (r_i) radius of the shaft
- The stacking combinations of the sheets that make up the composite

The outer and inner radii of the shaft are closely related to the number of blades by the expression:

$$r_o - r_i = h = t \cdot k \quad (2)$$

Where h represents the total thickness of the laminate, k the number of sheets and t the thickness of each sheet. The constraints of the optimization algorithm are given by a factor of safety based on a theory of failure, the natural frequencies of bending and torsion, and the critical torque due to torsional buckling.

Considering a large number of possible stacking orientations and sequences in the shaft laminate, a discretization of these variables is performed as suggested by Gürdal, Haftka, and Nagendra in their research [10].

To further facilitate the efficiency of the algorithm, only orientations at 0° , 90° , 45° , and -45° are taken into account, these last two orientations as successive pairs to guarantee their possibility of manufacture by the filament winding method [11]. Orientation selection is made considering that the normal stresses and ultimate shear stresses of the material are higher in these orientations.

Security factor

To ensure that the shaft is sufficiently resistant to the torsional loads applied, a safety factor equal to or greater than 3 is established [12]. A high value in the safety factor guarantees that all the sheets adequately resist the applied stresses, this due to the variation of the admissible stresses throughout the laminate.

In the particular case of design, the maximum shear stress is given by:

$$\tau_{m\acute{a}x} = \frac{T \cdot c}{J} \quad (3)$$

Where T represents the maximum torque, c the distance of the farthest fiber (outer radius) and J the polar moment of inertia.

Natural bending frequencies

A criterion of great importance in the design of rotors corresponds to the critical speed, that is, to the first bending mode, which must be sufficiently far from the maximum speed of rotation of the shaft, to avoid entering a state of resonance. To prevent the occurrence of this phenomenon, the value of the natural bending frequency is restricted above 1.5 [13] times the maximum speed of rotation.

Natural torsional frequencies

As with the natural frequencies of bending, the maximum speed of rotation of the shaft must not coincide and must be far enough away from the natural frequencies of torque of the shaft. To avoid the occurrence of this resonance phenomenon, the value of the natural torsion frequencies is restricted to above 1.5 [13] times the maximum speed of rotation.

Torsional buckling

Thin-walled tubular geometry shafts, that is, with slender geometry, can become buckled under the application of torsional loads. In order to avoid the collapse of the shaft during its operation, a critical torque is established, which is restricted to values equal to or greater than the maximum torque.

The critical torque in orthotropic composite materials [14] is given by:

$$T_{cr} = 2\pi r_m h \cdot 0,272 \cdot (E_x E_y^3)^{\frac{1}{4}} \left(\frac{h}{r_m}\right)^{\frac{3}{2}} \quad (4)$$

Where r_m is the mean radius and h the thickness of the shaft walls. E_x and E_y correspond to the longitudinal and transverse elastic moduli of the composite material.

Genetic algorithm setup

The most relevant configuration options for the genetic optimization algorithm [15] are:

- Population of 50 individuals
- 5 elite individuals
- A 0.9 crossover fraction (0.1 mutation)
- Infinite generations
- Successive generations without infinite changes
- Distance between the best solution and the average of all solutions of a generation.
- 23657 stacking combinations

Macroscopic analysis

Macroscopic analysis of a slide

To facilitate the analysis of the composite material, it is assumed that the sheets that make up the axis are orthotropic, homogeneous, elastic, of small thickness and that they do not present perpendicular loads. Under these considerations, the stress-strain relationship according to Hooke's general law [9] is:

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} \quad (5)$$

Where σ_1 and σ_2 are the longitudinal and transverse normal stresses, τ_{12} the shear stress in the principal plane, ϵ_1 and ϵ_2 the normal longitudinal and transverse deformations, and γ_{12} shear deformation in the principal plane. The values Q_{ij} represent the reduced stiffness coefficients and are given by:

$$Q_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}} \quad (6)$$

$$Q_{12} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}} \quad (7)$$

$$Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}} \quad (8)$$

$$Q_{66} = G_{12} \quad (9)$$

Where E_1 , E_2 are the longitudinal and transverse elastic constants, G_{12} the shear modulus in the main plane and, ν_{12} and ν_{21} the Poisson's ratios.

The matrix of reduced stiffness coefficients established in equation (5), is valid only for the particular orientations of the fibers in each sheet, however, for the analysis of the composite material the global orientations of the laminate (x, y, z) are more useful.

To determine the stress-strain relationship in the global axes of the laminate, the Reuter matrix [16] and a transformation matrix are used, which relates the local orientations of each sheet with the global orientations of the laminate, which allows obtaining the next relationship:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} \quad (10)$$

σ_x , σ_y and τ_{12} are the longitudinal and transverse normal stresses and the shear stress in the principal plane, in the global directions. ϵ_x , ϵ_y and γ_{xy} are the longitudinal, transverse normal strains and the principal plane shear strain, in the overall directions of the laminate. The elements of the transformed stiffness matrix \bar{Q}_{ij} are defined by:

$$\bar{Q}_{11} = Q_{11} \cos^4 \theta + 2(Q_{12} + 2Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{22} \sin^4 \theta \quad (11)$$

$$\bar{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{12}(\sin^4 \theta + \cos^4 \theta) \quad (12)$$

$$\bar{Q}_{16} = (Q_{11} - Q_{12} - 2Q_{66}) \sin \theta \cos^3 \theta + (Q_{12} - Q_{22} + 2Q_{66}) \sin^3 \theta \cos \theta \quad (13)$$

$$\bar{Q}_{22} = Q_{11} \sin^4 \theta + 2(Q_{12} + 2Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{22} \cos^4 \theta \quad (14)$$

$$\bar{Q}_{26} = (Q_{11} - Q_{12} - 2Q_{66}) \sin^3 \theta \cos \theta + (Q_{12} - Q_{22} + 2Q_{66}) \sin \theta \cos^3 \theta \quad (15)$$

$$\bar{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{66}(\sin^4 \theta + \cos^4 \theta) \quad (16)$$

θ is the angle of orientation of the reinforcing fibers with respect to the global orientations of the laminate.

Macroscopic analysis of the laminate

As a reference for the analysis, the coordinate system for the laminate is established in figure 2. The terms h_k represent the distance of the outer face of each sheet, with respect to the median plane of the laminate.

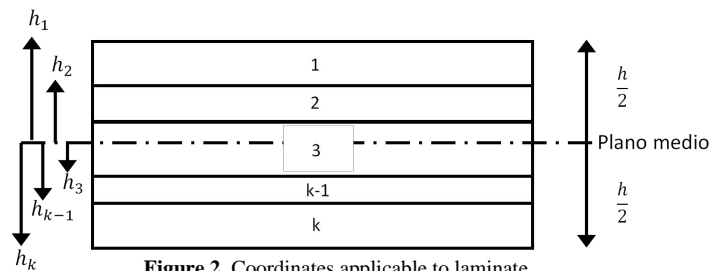


Figure 2. Coordinates applicable to laminate
Source: self made

To determine the elastic constants of the laminate, it is necessary to establish the stiffness matrices of extension A, coupling B, and bending D, which relate the forces and moments per unit length applied in the principal plane, with the deformations (ϵ_x^o , ϵ_y^o , γ_{xy}^o) and curvatures in the median plane (κ_x , κ_y , κ_{xy}). The matrices are expressed as a unique matrix [17] given by:

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{bmatrix} \epsilon_x^o \\ \epsilon_y^o \\ \gamma_{xy}^o \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \quad (17)$$

The elements of the matrix of extension A, coupling B and bending D, established in equation (17) are given by:

$$A_{ij} = \sum_{n=1}^k [\bar{Q}_{ij}]_k (h_k - h_{k-1}) \quad (18)$$

$$B_{ij} = \frac{1}{2} \sum_{n=1}^k [\bar{Q}_{ij}]_k (h_k^2 - h_{k-1}^2) \quad (19)$$

$$D_{ij} = \frac{1}{3} \sum_{n=1}^k [\bar{Q}_{ij}]_k (h_k^3 - h_{k-1}^3) \quad (20)$$

In the particular case of design, there is only one force per unit length corresponding to N_{xy} and is given by:

$$N_{xy} = \tau_{m\acute{a}x} \cdot h \quad (21)$$

The elements of the inverse matrix of the extension matrix A, denoted as a_{ij} , allow the calculation of the engineering constants of the laminate and are given by:

$$E_x = \frac{1}{h \cdot a_{11}} \quad (22)$$

$$E_y = \frac{1}{h \cdot a_{22}} \quad (23)$$

$$G_{xy} = \frac{1}{h \cdot a_{66}} \quad (24)$$

$$v_{xy} = \frac{a_{12}}{a_{11}} \quad (25)$$

Tsai-Wu Failure Criterion

The Tsai-Wu failure theory is the most widely implemented criterion in orthotropic composite materials, due to the close similarity of the theoretical results with the experimental results obtained in various studies[18], [19].

The Tsai-Wu theory [20] states that a sheet fails if:

$$H_1\sigma_1 + H_2\sigma_2 + H_6\tau_{12} + H_{11}\sigma_1^2 + H_{22}\sigma_2^2 + H_{66}\tau_{12}^2 + 2H_{12}\sigma_1\sigma_2 < 1 \quad (26)$$

The components H de Tsai-Wu theory are related to the ultimate longitudinal tensile stresses σ_{1u}^t and transversal σ_{2u}^t , and ultimate longitudinal compression stresses σ_{1u}^c and transversal σ_{2u}^c of the material, and are given by:

$$H_1 = \frac{1}{\sigma_{1u}^t} - \frac{1}{\sigma_{1u}^c} \tag{27}$$

$$H_2 = \frac{1}{\sigma_{1u}^t} - \frac{1}{\sigma_{1u}^c} \tag{28}$$

$$H_6 = 0 \tag{29}$$

$$H_{11} = \frac{1}{\sigma_{1u}^t \sigma_{1u}^c} \tag{30}$$

$$H_{22} = \frac{1}{\sigma_{2u}^t \sigma_{2u}^c} \tag{31}$$

$$H_{12} = -\frac{1}{2} \sqrt{\frac{1}{\sigma_{1u}^t \sigma_{1u}^c \sigma_{2u}^t \sigma_{2u}^c}} \tag{32}$$

Results and Discussion

Optimization algorithms are prone to be trapped in local minima [21], that is, providing an optimal solution for a subregion of the limits of the objective function, but not for the optimal solution of the entire domain of possible solutions. To avoid this particularity, the optimization algorithm is executed several times, obtaining a single solution, which allows inferring that the solution is optimal for the entire domain of possible solutions. The results obtained in an intermediate step of the optimization process and the final results of the optimization algorithm are shown in table 2 and table 3 respectively.

Table 2. Results obtained in an intermediate step of the execution of the optimization algorithm.

Results of the optimization algorithm	
Inner radius	18.2436 [mm]
Outer radius	20.7436 [mm]
Thickness	2.5 [mm]
No. of layers	10
Orientations	[45,-45,90,-45,45,45,-45,90,-45,45]
Weight	3.07311 [N]
Mass	0.313263 [kg]
Fact. of sec. Tsai-Wu	3.0155
Freq. natural bending	10111.5 [RPM]
Freq. natural torsion	59950.9 [RPM]
Fact. sec critical torque	1.18343

Source: Self-made

Table 3. Final results obtained from the execution of the optimization algorithm.

Results of the optimization algorithm	
Inner radius	18.5454 [mm]
Outer radius	20.5454 [mm]
Thickness	2 [mm]
No. of layers	8
Orientations	[45,-45,45,-45,-45,45,-45,45]
Weight	2.46501 [N]
Mass	0.251276 [kg]
Fact. of sec. Tsai-Wu	2.999
Freq. natural bending	9552.69 [RPM]
Freq. natural torsion	65302.3 [RPM]
Fact. sec critical torque	2.0409

Source: Self-made

The behavior of the optimization algorithm for the first two executions can be seen in Figures 3 and 4. The figures compare the objective function evaluated for its minimum value (black points) and its average value (green points) on the y axis, with the generations in which the genetic operators were applied on the x-axis.

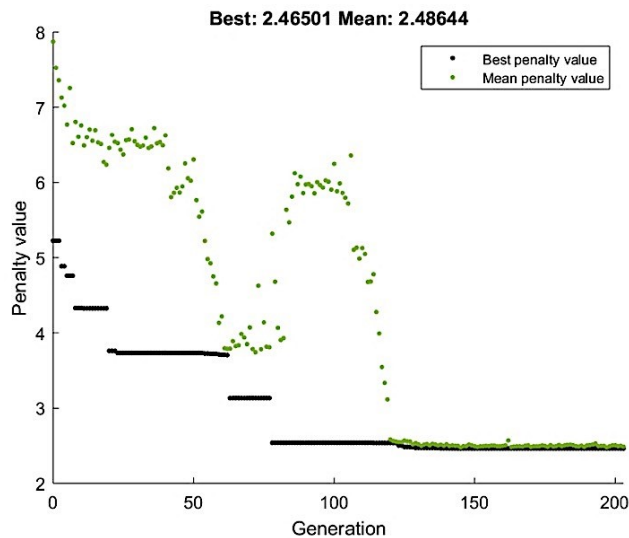


Figure 3. Evaluation of the objective function (best value and average) for execution 1. Source: own elaboration.

In the figures, it can be seen that the algorithm actually went through local minima (stepped jumps in the black points), however, it managed to converge in a unique solution for the two executions, whose value corresponding to the weight (objective function) is 2.4650 [N].

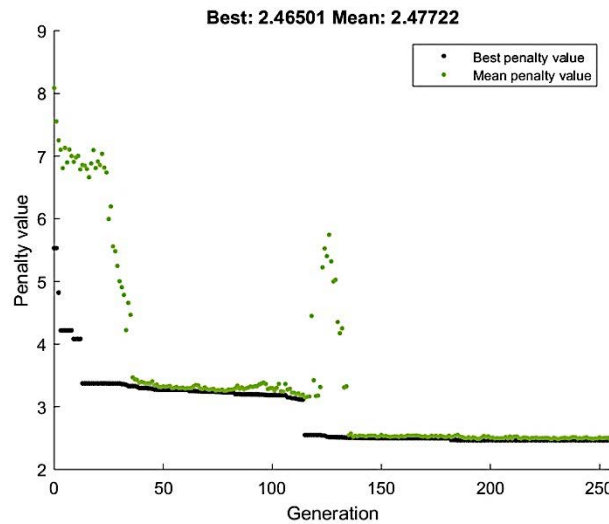


Figure 4. Evaluation of the objective function (best value and average) for execution 2. Source: own elaboration.

When all the fibers are oriented in the same direction and assuming the same geometric configuration obtained from the optimal shaft design, the following results are obtained:

Table 4. Results obtained from the variation of the orientation of the fibers in a single direction (proportion of 100%).

Orientations				Properties			
0°	90°	45°	-45°	FS	f_{nf}	f_{nt}	T_{cr}
100%	0	0	0	0,623	22291	30216	661
0	100%	0	0	0,623	5390	30216	2732
0	0	100%	0	2,11	7464	29780	623
0	0	0	100%	0,362	7464	29780	623

Source: self-made

When setting the laminate in a balanced way, that is, with the orientations of 0°, 90°, 45°, and -45° in the same proportions and symmetrically, the following results are obtained:

Table 5. Balanced and symmetrical lamination results.

Orientations				Properties			
0°	90°	45°	-45°	FS	f_{nf}	f_{nt}	T_{cr}
25%	25%	25%	25%	1,766	14067	50879	2213

Source: self-made

It is important to highlight that the results in table 5 correspond to 4 different stacking combinations that yielded the same results. The stacking combinations analyzed were:

- [90, 0, 45, -45, -45, 45, 0, 90]
- [0, 45, -45, 90, 90, -45, 45, 0]
- [45, -45, 90, 0, 0, 90, -45, 45]
- [-45, 90, 0, 45, 45, 0, 90, -45]

With the results, it can be inferred that, in symmetric and balanced laminates, the properties of the material do not depend on the stacking sequence, but only on the proportion of the orientations.

By maintaining the same proportion of orientations, but this time with a non-symmetric stacking sequence, such as [0, -45, 90, 45, -45, 0, 45, 90] we obtain the following results:

Table 6. Balanced and non-symmetrical rolling results.

Orientations				Properties			
0°	90°	45°	-45°	FS	f_{nf}	f_{nt}	T_{cr}
25%	25%	25%	25%	1,363	14067	50879	2213

Source: self-made

This new configuration maintains the critical torque and the natural frequencies of bending and torsion equal to those obtained in table 5, but the safety factor changes significantly.

Conclusions

- From the results obtained in the development of this analysis, it can be concluded that both the orientations and the stacking sequence in laminated composite materials play a role of great importance and are conditioned by design restrictions.
- The longitudinal orientations (0 °) concerning the global coordinates in the laminate, increase the stiffness and consequently increase the natural frequencies of bending.
- The transversal orientations (90 °) concerning the global coordinates in the laminate prevent a slender axis, that is, with tubular geometry of small thickness concerning its diameter, from collapsing due to the phenomenon of torsional buckling.
- Orientations at 45 ° and -45 ° provide the best response to torsional loads, however, it is important to consider the orientation in which the fibers are arranged to ensure maximum strength. As can be seen in the safety factor in table 4, the direction of the angle plays an important role. That is, orienting the fibers in the same direction of application of the load, provides the greatest resistance, on the other hand, orienting them in the opposite direction, notably affects a lower resistance.
- Combining various orientations in the laminate of composite material provides the material resistance to various load and operating conditions.
- The stacking sequence in non-symmetrical laminates has a direct impact on the safety factor and consequently on the loads that can be applied to the composite material.

References

- [1] P. Těšínová, “Advances in Composite Materials - Analysis of Natural and Man-Made Materials”, *InTech*, 2012.
- [2] M. Kutz, “Handbook of Materials Selection”, J. Wiley, 2007.
- [3] J. R. Duflou, J. De Moor, I. Verpoest and W. Dewulf, “Environmental impact analysis of composite use in car manufacturing”, *CIRP Annals Manufacturing Technology*, vol. 58, no. 1, pp. 9-12, 2009.
- [4] D. Calestani, M. Villani, M. Culiolo, D. Delmonte, N. Coppedè and A. Zappettini, “Smart composites materials: A new idea to add gas-sensing properties to commercial carbon-fibers by functionalization with ZnO nanowires”, *Sensors and Actuators B: Chemical - Journal*, vol. 245, pp. 166-170, 2017.
- [5] M. Petrovic, T. Nomura, T. Yamada, K. Izui and S. Nishiwaki, “Orthotropic material orientation optimization method in composite laminates”, *Structural and Multidisciplinary Optimization*, vol. 57, no. 2, pp. 815-828, 2018.
- [6] H. G. Sánchez Acevedo, J. Uscátegui and S. Gómez, “Metodología para la detección de fallas en una estructura entramada metálica empleando las técnicas de análisis modal y PSO”, *Revista UIS Ingenierías*, vol. 16, no. 2, pp. 43-50, 2018.

- [7] C. M. Escobar, O. A. González-Estrada and H. G. S. Acevedo, "Damage detection in a unidimensional truss using the firefly optimization algorithm and finite elements", *arXiv Prepr. arXiv1706.04449*, 2017.
- [8] L. Araujo and C. Cervigón, "Algoritmos evolutivos: un enfoque práctico", *Ra-Ma*, 2009.
- [9] A. K. Kaw, "Hooke's Law for a Two-Dimensional Unidirectional Lamina", *Mechanics of Composite Materials*, 2nd ed., Boca Raton: Taylor & Francis, pp. 466, 2006.
- [10] S. Nagendra, Z. Gurdal and R. T. Haftka, "Stacking sequence optimization of simply supported laminates with stability and strain constraints", *AIAA Journal*, vol. 30, no. 8, pp. 2132-2137, 1992.
- [11] S. C. Mantell and D. Cohen, "Filament Winding", *Processing of Composites*, pp. 388-417, 2012.
- [12] M. A. K. Chowdhuri and R. A. Hossain, "Design Analysis of an Automotive Composite Drive Shaft", 2010.
- [13] J. C. Leslie, L. Troung, J. C. Leslie, B. Blank and G. Frick, "Composite Driveshafts: Technology and Experience", *SAE Technical Paper Series*, vol. 1, pp. 404-413, 2010.
- [14] N. C. Kenkyū Iinkai, "Column Research Committee of Japan", *Handbook of structural stability*, Tokyo: Corona Publ. Co., 1971.
- [15] MathWorks, "Find minimum of function using genetic algorithm - MATLAB ga", *Matlab documentation*, 2018.
- [16] B. John, L. Andrew and R. Carl, "Mechanics of Solids", *Abingdon: Routledge*, 2 ed., 2016.
- [17] A. K. Kaw, "Macromechanical Analysis of Laminates", *Mechanics of Composite Materials*, 2nd ed., pp. 315-367, 2010.
- [18] P. Nali and E. Carrera, "A numerical assessment on two-dimensional failure criteria for composite layered structures", *Composites Part B: Engineering*, vol. 43, no. 2, pp. 280-289, 2012.
- [19] I. M. Daniel, J. J. Luo, P. M. Schubel and B. T. Werner, "Interfiber/interlaminar failure of composites under multi-axial states of stress", *Composites Science and Technology*, vol. 69, no. 6, pp. 764-771, 2009.
- [20] W. Tsai, Stephen and M. Wu, Edward, "A General Theory of Strength for Anisotropic Materials, Composite Materials", *Journal of Composite Materials*, vol. 5, no. 1, pp. 58-80, 1971.
- [21] Y. T. Kao and E. Zahara, "A hybrid genetic algorithm and particle swarm optimization for multimodal functions", *Applied Soft Computing*, vol. 8, no. 2, pp. 849-857, 2008.