



## Application of Finite Elements for the Check of Hydraulic Concrete Slabs

Aplicación de Elementos Finitos para el Chequeo de Losas de Concreto Hidráulico

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**How to cite:** C.H. Higuera-Sandoval. D.A. Hernández-Rojas, “Application of Finite Elements for the Check of Hydraulic Concrete Slabs.” *Respuestas*, vol. 26, no. 2, pp. 14-26, 2021.

Received on February 07, 2020; Approved on April 5, 2020

### ABSTRACT

#### Keywords:

Deflections, Segments,  
Finite Elements, Efforts,  
EverFe, Concrete slabs,  
Rigid pavement.

Since the human being had the need to move from one place to another quickly and reliably, the pavement is created in order to facilitate the transport of people and goods, deriving various types of structures with their own characteristics. Rigid pavement, due to its properties, has a greater participation in road construction since it provides high resistance in bearing loads, greater durability and little maintenance. Technological development optimizes study, design, and construction processes by offering tools that include the use of finite elements, such as EverFe 2.26. This software allows to analyze of a rigid pavement structure composed of subgrade, granular subbase, and hydraulic concrete slabs, determining forces and deflections caused by friction factors, warping, and traffic loads; that, for the object and methodology of the study, two types of scenarios are formulated: 1. Analysis of design parameters for only one slab and 2. Analysis of design parameters for a set of slabs using segments and tie bars. The results given will be a point of comparison to determine maximum efforts and deflections in both scenarios, directly obtaining the justification of the important use of load transmission elements and the essentials of the application of finite elements when designing rigid pavement structures, since the behavior of the same is studied in depth obtaining real results of greater precision.

### RESUMEN

#### Palabras clave:

Deflexiones, Dovelas,  
Elementos Finitos,  
Esfuerzos, EverFe, Losas  
de concreto, Pavimento  
rígido.

se crea el pavimento con la finalidad de facilitar el transporte de personas y mercancías, derivándose varios tipos de estructuras con características propias. El pavimento rígido por sus propiedades, tiene mayor participación en la construcción vial ya que proporciona alta resistencia en el soporte de cargas, mayor durabilidad y poco mantenimiento. El desarrollo tecnológico optimiza procesos de estudio, diseño y construcción del mismo ofreciendo herramientas que incluyen el uso de los elementos finitos como, por ejemplo, EverFe 2.26. Este software permite analizar una estructura de pavimento rígido compuesta por subrasante, subbase granular y losas de concreto hidráulico determinando esfuerzos y deflexiones ocasionados por factores de fricción, alabeo y cargas de tránsito; que, para el objeto y metodología de estudio, se formulan dos tipos de escenario: 1. Análisis de Parámetros de diseño para solo una losa y 2. Análisis de parámetros de diseño para un conjunto de losas haciendo uso de las dovelas y barras de amarre. Los resultados dados, serán punto de comparación para determinar esfuerzos y deflexiones máximas en ambos escenarios, obteniendo directamente la justificación del importante uso de los elementos de transmisión de carga y lo esencial de la aplicación de elementos finitos a la hora de diseñar estructuras de pavimento rígido, ya que se estudia a fondo el comportamiento del mismo obteniendo resultados reales de mayor precisión.

### Introduction

The finite element modeling of rigid pavement structures has become more important in recent years due to the high technological progress since it allows us to know more accurately the real behavior of a road structure under loading conditions and environmental effects [1]. This method is based on the numerical analysis for the determination of stresses and deflections utilizing the discretization of the element connected through nodes and arcs, where the accuracy of the results is directly proportional to the size of the mesh with which it works; that is, the smaller the discretization or division of the element, the more accurate the results will be.

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EverFe is one of the software based on three-dimensional finite element models, developing its usefulness in the field of study and design of rigid pavements because it guarantees a more accurate approach in obtaining results of stresses and deflections in slabs [2], offering a wide range of analysis scenarios where you can take into account the longitudinal and transverse segments, joints, load spectra and modeling of one or more slabs [3]; making it a more practical and complete method concerning analytical methodology software.

Through EverFe 2.26, a rigid pavement structure is modeled corresponding to an urban road with hydraulic concrete slabs, sub-base support layer following INVIAS guidelines and subgrade. This structure is subjected to loads generated by vehicles and external factors of temperature changes that allow us to see the behavior of the structure against factors that cause stresses and deflections in the slabs. For this, two scenarios are applied: 1) Modeling a slab with a support layer and 2) Modeling a group of slabs where the presence of voussoirs and tie bars are present, looking together at the indispensable function that this type of reinforcement has in the pavement; since these elements are responsible for the load transmission and durability of the joint system, directly influencing the performance of rigid pavement, where if the transmission is inadequate, the slabs would be affected since the load capacity is reduced, directly causing structural damage to the pavement.

By determining the factors that influence the behavior of the structure, it is intended to have a basic knowledge about the design procedure of rigid pavement, demonstrating whether the structure is suitable or not for the Ndesign calculated in the traffic study. In this way, it is shown in a summarized and concrete way the optimal methodology at the time of defining each one of the efforts and deformations, the treatment of the obtained information, the correct check of the slabs by criteria given by AASHTO and INVIAS, and the analysis of results; all applied to a real structure using the finite element modeling.

## **Materials and Methods**

### ***Analysis of the finite element method***

As technology advances, this method has become more useful in the modeling of rigid pavement structures because it can accurately represent the field conditions of traffic loading and weathering. It is a method that is highly dependent on the mesh size or number of element divisions and boundary conditions. Previous research has defined that the optimum number of divisions should be  $16*16*2$  [4] [5]; however, this criterion is not universal since other studies have established a division of  $32*32*3$  according to the dimensions of the slab [3]. It should be noted that the mesh used in this modeling is of equal size in each layer of the pavement structure, in addition to greatly affecting the accuracy of the results, i.e., having a smaller mesh the results will be more accurate. However, if the mesh is too small it leads to more time and computational work [3], so, for this article, several modeling was performed by varying the mesh width between 16 to 32, establishing that, to have good accuracy of the information in a short modeling time and low memory capacity, the meshing of  $24*24$  in the (x-y) plane and a subdivision in the z-plane of 3 elements is applied.

To have an overview of the meshing work, it is taken into account that the geometry of the model is rectangular, where EverFE handles a 20-node cube with rectilinear meshes for the upper layers and an 8-node rectangle for the subgrade as shown in Figure 1.

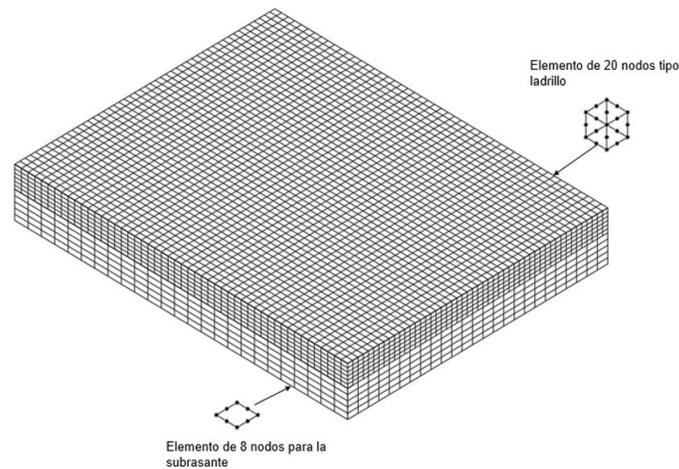


Figure 1. EverFe framing.

The finite elements use constitutive models (elastic, plastic, elastoplastic, viscoelastic, etc.) to determine the stresses, deformations, and deflections in a micromechanical way; that is to say, at a global level in fine, granular treated, and untreated materials. In the static analysis, the material properties must be taken into account, which is expressed in a stiffness matrix ( $K$ ) and column vectors of displacements or unknowns ( $u$ ) and external forces ( $F$ ) that intervene in the problem domain. See equation 1.

$$[K]\{u\} = \{F\} \quad (1)$$

The implementation of this method is very useful since it generates color diagrams that offer a broader view of the stresses and deflections of the slab at the points of interest such as edge, corner, and interior of the slab, which can be divided according to their cause into transit, warping and friction stresses [6][7][7][8][9].

### *Analysis of the empirical-mechanistic method*

This method was developed by Westergaard (1948) which considers a free slab applying a single wheel load with a circular contact area to determine the stresses and deflections at critical points of interest such as edges, corners, and interior of the slab. For each point of interest, there is a particular equation that allows knowing the stress states of the structure at the mentioned points; likewise, this method has the limitation that these results can only be known at a specific point, unlike the finite element method, which offers a global view of the behavior of the structure against external loads and the action of the weather. [10][11] [12].

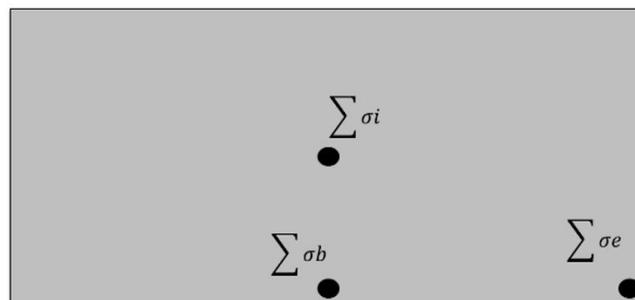


Figure 2. Study points for slab stresses.

The sum of stresses at each of the critical points can be broken down in equations 2, 3 and 4 as follows:

$$\Sigma\sigma_{interior} = \sigma_{friction} + i_{alabeo}\sigma_{(critical)} + \sigma_{transit} \quad (2)$$

$$\Sigma\sigma_{edge} = \sigma_{friction} + \sigma_{(critical)} \sigma_{warbling} + \sigma_{transit} \quad (3)$$

$$\Sigma\sigma_{corner} = \sigma_{friction} + \sigma_{alabeo} \sigma_{(critical)} + \sigma_{transit} \quad (4)$$

For the determination of the stress states of the structure, the EverFe 2.26 program was used.

### Characterization of axles

The traffic study is an indispensable requirement when designing a pavement structure, since it projects and determines the number of 8.2 ton equivalent axles that will pass through the roadway, constituting the -Ndesign, defining by classification, the traffic level with which the thickness of the slab is calculated using the INVIAS methodology [13]. For this article, the structure is analyzed with real data from the traffic study provided by the Mayor's Office of Motavita-Boyacá, where for the paving project of 4th Street in hydraulic concrete, it shows average daily traffic - TPD of 656 vehicles. Following the INVIAS procedure [13], the design traffic-Ndesign corresponding to NT2 is calculated, where 8% belongs to commercial vehicles up to the classification of 3S3 (design vehicle).

The number of tires and weight per axle for the design vehicle is determined according to the current resolution 4100 of 2004, which regulates the type of vehicles, dimensions, maximum gross weight, and axle type of motor vehicles for land transport [14], as shown in Table I.

Table I. Axles and Weight Per Axle for 3s3 Truck.

Vehicle Section	Axle Type	No. Tires	Axle Weight
			[Kg]
Headband	Single directional	2	6000
	Tandem	8	22000
Semi tráiler	Tridem	12	24000

Source: Authors. Taken and Edited from the Ministry of Transport. Resolution 4100 of 2004. Colombia.

For its respective modeling, Figure 3 shows a planning scheme where the information is given in Table I is graphically represented, where the separation of the axles in the X and Y direction is taken into account. To get results closer to reality, measurements of axle spacing and tire width are taken to determine the geometry of the load plate or contact area of each of the axles. In this way, it is possible to model them individually using the load spectrum methodology to determine the damage caused in the slab.

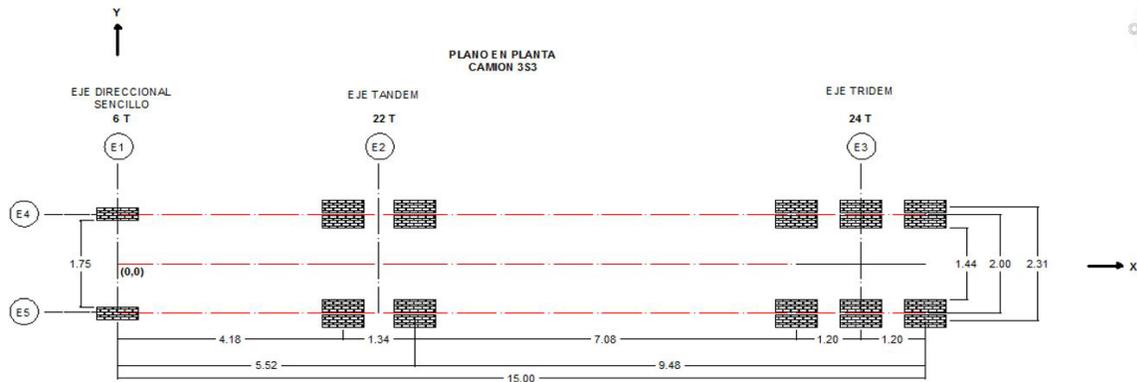


Figure 3. 3S3 truck layout.

### Load footprint

For the modeling of axles, EverFe requires the definition of the geometry of the contact area between square and rectangular to consider each tire separately; in this way, the axle weight is distributed by the number of tires that compose it to know the load per tire that will be transmitted directly to the concrete slab. For the case study, a rectangular footprint is assumed and its dimensions are shown in Table II.

Table II. Contact Area By Shaft Type.

Axle Type	No. Tires	Axle Weight		Contact pressure [Kg/cm <sup>2</sup> ]	Contact Area-Ac [cm <sup>2</sup> ]	Footprint length - Lf [cm]	Width of the W-track [cm]
		[Kg]	[Kg]				
Single directional	2	6000	3000		416.67	24.60	16.94
Tandem	8	22000	2750	7.2	381.94	23.55	16.22
Tridem	12	24000	2000		277.78	20.08	13.83

### Structural Model

The pavement structure to be modeled is obtained with the INVIAS methodology [13] from the following design parameters:

- Subgrade CBR of 5.5%, subgrade categorization S3.
- Granular sub-base layer according to the specifications of Article 320 of the INVIAS-2013 with a thickness of 25 cm class B.
- Hydraulic concrete slab with a modulus of rupture of 40 kg/cm<sup>2</sup>.
- Diseño NT2 that a hydraulic concrete slab thickness of 21 cm with dimensions 320 cm wide and 250 cm long is obtained.
- Temperature gradient - ΔT. It indicates the temperature difference that the slab undergoes due to climatic changes causing differential contractions or movements throughout the slab [15].

- Modulus of reaction of the whole - Keffective. This is a determining factor in the design of a rigid pavement because it encompasses the reaction of the layers underlying the slab, including the subgrade. It is known that the slab cannot be placed directly on the foundation soil, due to several properties that affect it, so it requires overcoming this problem by utilizing the proportion of layers between the subgrade and the slab so that the stresses generated by traffic can be received and transmitted to the subgrade in a smaller proportion [4].
- Load transfer system with 1 1/8 inch diameter [2.86 cm] voussoirs, 40 cm long and 30 cm apart, and with 1/2 inch diameter [1.27 cm] tie bars, 60 cm long and 100 cm apart.

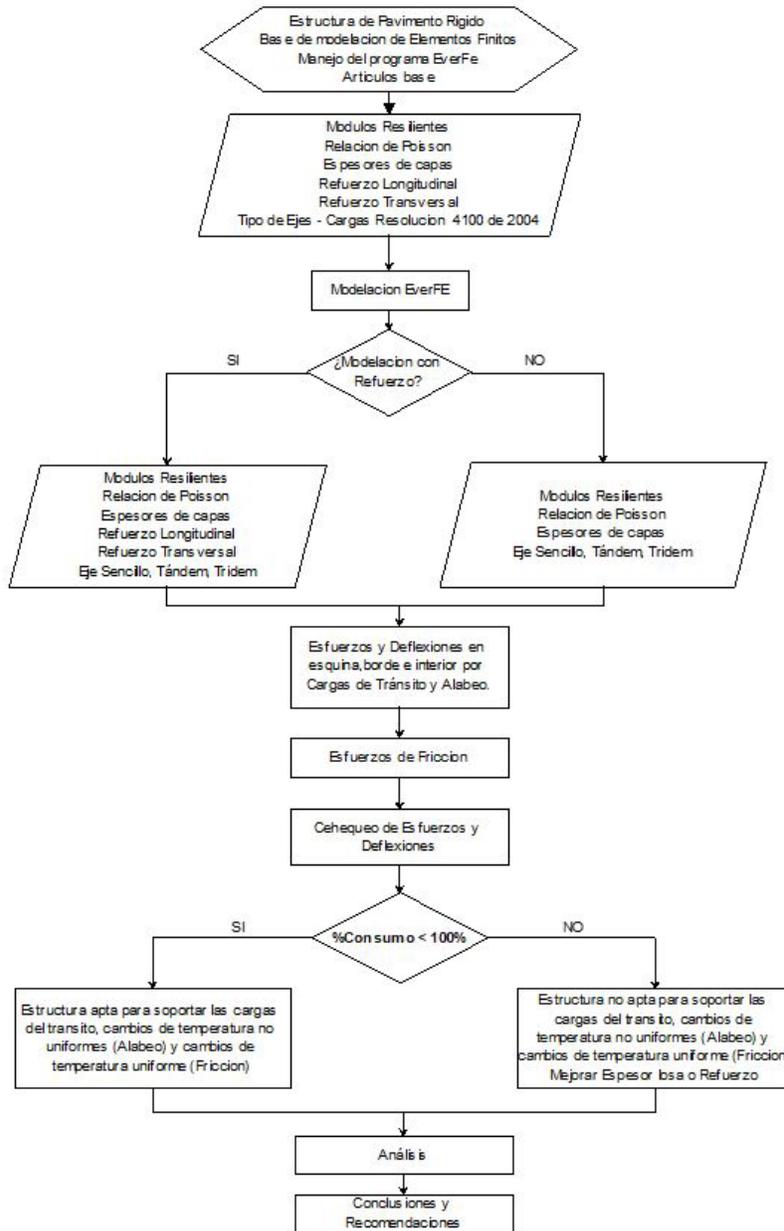


Figure 4. shows the Flow Chart that summarizes the methodology used in this research.

## Results and Analysis

When modeling the rigid pavement structure, the points to be evaluated (interior, edge, and corner) of the slab are defined by determining stresses in the X-Y direction and deflections at each of the analysis points. These results are defined by axis type in Table III.

Table III. Stresses and Deflections are Generated by the Type Of Shaft.

Axle Type	Point Evaluated	Service Values [Kg/cm <sup>2</sup> ].		Point Evaluated	Service Values	
		Transit	Praise		[cm]	
Directional Single	Corner [ $\sigma_e$ ]	12.32	5.580	Corner	$\Delta_e$	0.0752
	Edge [ $\sigma_b$ ]	11.29	10.91	Edge	$\Delta_b$	0.0482
	Interior [ $\sigma_i$ ]	9.840	10.94	Inside	$\Delta_i$	0.0228
Tandem	Corner [ $\sigma_e$ ]	18.04	5.580	Corner	$\Delta_e$	0.1226
	Edge [ $\sigma_b$ ]	13.21	10.91	Edge	$\Delta_b$	0.0790
	Interior [ $\sigma_i$ ]	9.910	10.94	Inside	$\Delta_i$	0.0412
Tridem	Corner [ $\sigma_e$ ]	14.11	5.580	Corner	$\Delta_e$	0.0589
	Edge [ $\sigma_b$ ]	12.82	10.91	Edge	$\Delta_b$	0.0586
	Interior [ $\sigma_i$ ]	8.150	10.94	Inside	$\Delta_i$	0.0327

In addition to this, the frictional forces generated by uniform temperature changes ( $\sigma_f$ ) must be determined. This is determined by equation 5:

$$\sigma_f = 0.18 * L \quad [Kg/cm^2] \quad (5)$$

Where:

L = Length of the slab [m].

The calculated value is 0.45 Kg/cm<sup>2</sup>, which is the same for all the evaluated points and for each type of modeled axis since it is only a function of the length of the slab.

With  $\sigma_f$  calculated, a check of the corner, interior, and edge stresses are performed using equations 2, 3, and 4, verifying that it complies with the design criteria. In the same way, a deflection check is performed where none of the obtained deflections is greater than 0.15 cm being this the maximum value for the failure of the concrete.

Table IV. Effort Check.

Axle Type	Point Evaluated	Service Values	Allowable Values	Consumption	Observation
		[Kg/cm2]	[Kg/cm2]		
Directional Single	$\sigma_e$	23.69		56.39	Complies
	$\sigma_b$	22.66	42	53.94	Complies
	$\sigma_i$	21.24		50.57	Complies
Tandem	$\sigma_e$	29.41		70.02	Complies
	$\sigma_b$	24.57	42	58.51	Complies
	$\sigma_i$	21.31		50.74	Complies
Tridem	$\sigma_e$	25.48		60.67	Complies
	$\sigma_b$	24.19	42	57.58	Complies
	$\sigma_i$	19.54		46.53	Complies

Table V. Deflection Check.

Axle Type	Point Evaluated	Service Values	Allowable Values	Consumption	Observation
		[cm]	[cm]		
Directional Single	$\Delta_e$	0.075		5.01	Complies
	$\Delta_b$	0.048	0.15	3.21	Complies
	$\Delta_i$	0.023		1.52	Complies
Tandem	$\Delta_e$	0.123		8.17	Complies
	$\Delta_b$	0.079	0.15	5.27	Complies
	$\Delta_i$	0.041		2.75	Complies
Tridem	$\Delta_e$	0.059		3.93	Complies
	$\Delta_b$	0.059	0.15	3.91	Complies
	$\Delta_i$	0.033		2.18	Complies

In the same way, the forces and deflections for the slab are determined considering the segments and tie bars, as shown in Table VI.

Table VI. Stress and Deflection Check of Slabs With Voussoirs and Tie Bars.

Axle Type	Point Evaluated	Service Values	Allowable Values	Consumption	Observation	
		kg/cm <sup>2</sup> , [mm], [kg/cm <sup>2</sup> ], [mm].	kg/cm <sup>2</sup> , [mm], [kg/cm <sup>2</sup> ], [mm].			
Directional	$\sigma_e$	37.249		88.69	Complies	
	$\sigma_b$	25.948	42	61.78	Complies	
	$\sigma_i$	25.402		60.48	Complies	
	Single	$\Delta_e$	0.029		1.94	Complies
		$\Delta_b$	0.028	0.15	1.87	Complies
		$\Delta_i$	0.027		1.82	Complies
Tandem	$\sigma_e$	41.054		97.75	Complies	
	$\sigma_b$	26.141	42	62.24	Complies	
	$\sigma_i$	23.494		55.94	Complies	
	Single	$\Delta_e$	0.035		2.30	Complies
		$\Delta_b$	0.036	0.15	2.37	Complies
		$\Delta_i$	0.036		2.39	Complies
Tridem	$\sigma_e$	27.050		64.40	Complies	
	$\sigma_b$	21.800	42	51.90	Complies	
	$\sigma_i$	20.320		48.38	Complies	
	Single	$\Delta_e$	0.030		2.01	Complies
		$\Delta_b$	0.028	0.15	1.88	Complies
		$\Delta_i$	0.029		1.96	Complies

As can be seen in Tables IV, V, and VI, two types of modeling were carried out. In the first one, a slab without segments or tie bars was considered and in the second one, the set of slabs with their respective segments and tie bars were considered; to which the stresses and deflections produced by traffic loads, warping, and friction were determined. According to the above, it is observed in the first modeling that the service values of the structure are lower than the admissible ones, having a maximum consumption percentage of 70% for the stresses and 81.73% for the deflections. Both values were produced by the tandem axis and were located at the corner of the slab. The stresses of the tridem axis were below the stress states produced by the tandem axis due to the dimensions of the slab, since it was not possible to locate the 3 axes within the same element, forcing to place the load only of the first 2 axes at the points of interest.

In the second modeling, it was observed that the service values of the structure, considering the load transfer and tie bars, are lower than the admissible ones, obtaining a maximum consumption value of 97.75% for the forces and 23.93% for the deflections. Unlike the first model, these percentages were obtained in different points, the maximum stresses were located in the corner of the most loaded slab and the deflections in the interior of the slab where both values were produced by the tandem axis.

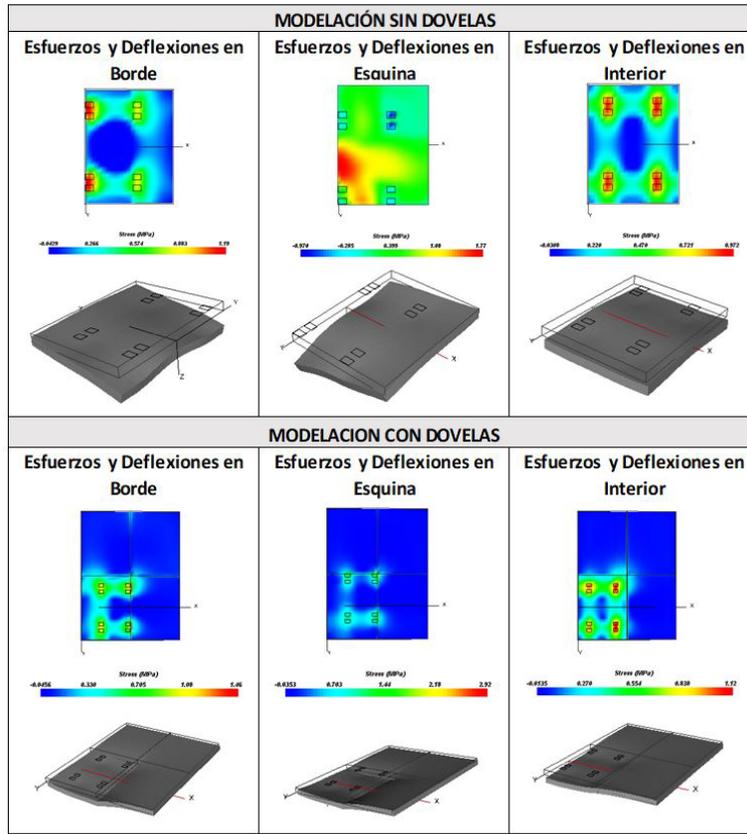
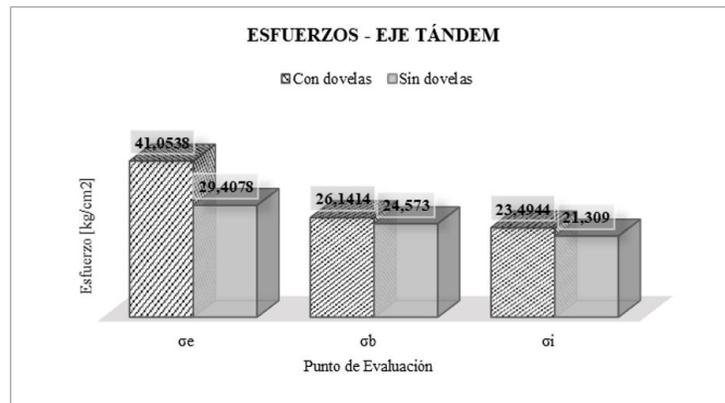
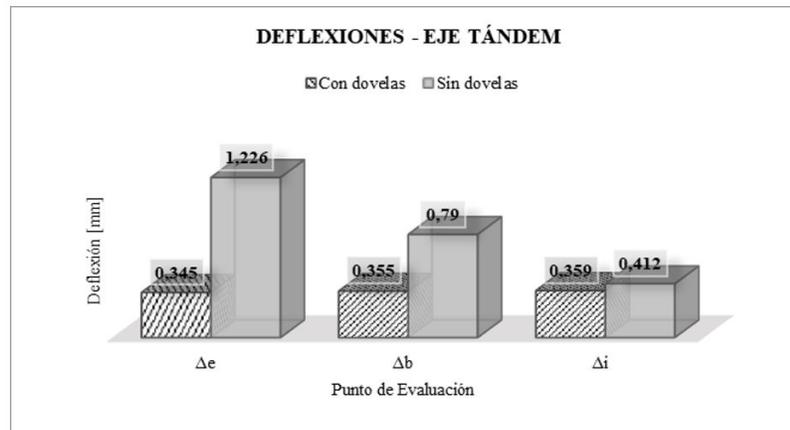


Figure 5. Tandem axle stresses and deflections - escenario 1 and 2.

In the two modeled scenarios, it could be observed that the tandem axis causes the highest stresses and deflections as shown in Figure V, which were located at the corner and inside the slab. It should be noted that in the second modeling the maximum deflection was obtained in the interior of the slab, which is improbable because at this point there is an adequate distribution of forces and deflections, deducing that the result is a consequence of the incorporation of the voussoirs and tie bars, which are responsible for transmitting the deflections to the attached slabs where the magnitude is greatly reduced at the edge and corner points. This decrease corresponds to 71.86% at the corner, 55.06% at the edge, and 12.86% inside the slab. In the second scenario, the stresses increased 28.35% at the edge, 6 % at the corner, and 9.30 % at the interior for the loaded slab and parallel to it.



Graph 1. Tandem axis forces for two scenarios.



Graph 2. Tandem axis deflections for two scenarios.

As can be seen in graph I, the stresses increase when the structure is modeled with segments, allowing to see the effective load transfer that this system generates in the slabs causing that the load-bearing work is done by the whole and not individually, observing that the most affected point is the corner having the highest magnitude of stresses.

If the change in magnitude of the deflections in the two scenarios is analyzed, graph II shows that the load transmitting elements considerably benefit the slabs, since they allow them to suffer less in terms of deformations due to the improvement of stress transfer, allowing the prevention of premature failure caused by traffic loads. As a complement, the segments and tie bars help to effectively and durably contain the movements of the element caused by non-uniform temperature changes, i.e. warping stresses.

These stresses are the ones that are most present in the slabs since even without a vehicle passing over it, it is already working alone due to the action of the thermal gradient ( $\Delta T$ ) that generates high temperature in the upper part of the slab and low in the lower part and vice versa, taking a concave or convex shape, a disposition that generates additional major stresses.

## Conclusions

When designing a rigid pavement, the procedure for obtaining input data must be carried out with caution, since it is these data that determine whether or not the structure complies with the calculated N design.

It is important to see that the contact area is one of the most representative factors when modeling in EverFe since it represents how the load can be transferred from the shafts to the slabs.

The finite element modeling allows obtaining results closer to reality, as it is reflected in the increase of the magnitude determined by EverFe in comparison with the manual analytical methods. This increase becomes a "safety factor" which, if it is not foreseen in the design process of the structure in the phase of checking the slabs, will cause an early failure of the slabs because the stresses caused by the level of traffic will be higher than estimated.

The three-dimensional analysis provided by EverFe allows for better visualization of the stresses and deflections generated in the slab. For this visualization to be more in line with reality, it requires that the results obtained are more accurate and in the need to optimize the collection of information, meshing plays a very important role. Making a relation of previous research on the modeling in this type of software, the division of the layers should be between 16 and 32 with an optimum of 24 in x - y, and 3 in z; since it is verified that the modeling of 24 onwards, the results of efforts and deflections become similar with the low difference between them because it is not justified to model with

32 divisions that demand more time and computational capacity if the same results will be obtained if it is modeled with 24 divisions.

It was found that the finite element modeling, allows to adequately represent the behavior of the segments and tie bars since the largest decrease was detected in the deflections with a reduction of about 71%. It shows that this load transfer system is very effective in helping the slab to adequately support the traffic loads.

By analyzing the stresses and deflections at the points of interest (corner, interior, and edge), it could be deduced that the section of the slab that suffers most from warping and traffic load effects is the corner. Therefore, this point is critical when mentioning the structural failures of the slabs since it is the most susceptible to stresses; causing this to be the initial point for failure, generating corner cracks, which is one of the most common pathologies in rigid pavements.

When the set of slabs was considered, the stress due to traffic load increased in all the evaluation points and in a more significant way in the corner showing the real behavior of the pavement; demonstrating that it is one of the most critical points, but less studied, so it is necessary to carry out more detailed investigations to explain this phenomenon. As the serviceability values in the two scenarios were less than the permissible values, it can be said that the structure will perform adequately and will adequately support the number of load repetitions during its entire service life. To have a more realistic idea of the behavior of the pavement structure, the serviceability values of the second scenario should be taken into account since this is the way the structure will be built and operated during its service life.

### Acknowledgements

The authors would like to thank the School of Transportation and Roads of the Faculty of Engineering of the Pedagogical and Technological University of Colombia for the support provided for this research.

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